Total No. of Printed Pages:02

SUBJECT CODE NO:- B-2161
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. F.Y (Sem.-II) Examination OCT/NOV 2019
Mathematics MAT - 201
(Integral Calculas)

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
1) Attempt all questions.
2) Figures to the right indicate full marks.

Q.1 **A)** Attempt any one: **08**

a) Obtain a reduction formula for $\int x^n e^{ax} dx$ and apply it to evaluate $\int x^3 e^{ax} dx$.

b) Obtain a reduction formula for $\int \cos^n x\, dx$, where n is positive integer. Hence evaluate $\int \cos^4 x\, dx$.

B) Attempt any one: **07**

c) Evaluate $\int \frac{2x-3}{(x^2-1)(2x+3)}\, dx$

d) Evaluate $\int \frac{(x^2+x+1)}{(x+1)^2(x+2)}\, dx$

Q.2 **A)** Attempt any one **08**

a) Evaluate $\int_a^b \sin x\, dx$ as the limit of a sum.

b) Find the area enclosed between one arch of the cycloid $x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$ and its base.

B) Attempt any one **07**

c) Find the length of the arc of the curve $y = \log \sec x$ from x=0 to x=$\pi/3$.

d) Find the volume of the solid obtained by revolving the cardioide $r = a(1 + cos\theta)$ about the initial line.

Q.3 A) Attempt any one **05**

a) Show that the volume obtained by revolving about X-axis, the arc of the curve $y = f(x)$, intercepted between the points whose abscissae are a,b is $\int_a^b \pi y^2 dx$, it being assumed that the arc does not cut X-axis.

b) Prove that the necessary and sufficient condition for a continuous vector point function to be irrotational in a simply connected region R is that it is the gradient of a scalar point function.

B) Attempt any one: **05**

c) Evaluate $\int_C \vec{F}.d\vec{r}$ when

$$\vec{F} = xy\vec{\imath} + yz\vec{\jmath} + zx\vec{k}$$

Where C is the curve
$r = \vec{\imath}t + \vec{\jmath}t^2 + \vec{k}t^3$; t varying from -1 to +1.

d) If $\overrightarrow{OA} = a\vec{\imath}$, $\overrightarrow{OB} = a\vec{\jmath}$, $\overrightarrow{OC} = a\vec{k}$ form three coterminous edges of a cube and S denotes the surface of the cube,
Evaluate $\int_S \{(x^3 - yz)\vec{\imath} - 2x^2y\vec{\jmath} + 2\vec{k}\}$. nds

Q.4 Choose the correct alternative and fill in the blanks. **10**

1) $\int \frac{dx}{(3-2x)^4} = -----$

$a) -\frac{1}{6}\cdot\frac{1}{(3-2x)^3}$ b) $\frac{1}{6}\cdot\frac{1}{(3-2x)^3}$ c) $\frac{1}{(3-2x)^3}$ d) $\frac{1}{6(3-2x)^5}$

2) $\int \cos^3 x\, dx = ------$

a) $-\sin x + \frac{\sin^3 x}{3}$ b) $-\sin x - \frac{\sin^3 x}{3}$
c) $\sin x - \frac{\sin^3 x}{3}$ d) $\sin x + \frac{\sin^3 x}{3}$

3) A curve $\vec{r} = \vec{f}(t)$, is called smooth if $\vec{f}(t)$, is ------------.
a) Differentiable
b) Continuously differentiable
c) Discontinuous
d) None of these

4) The process of determining the area of a plane region is known as ---------.
a) Quadrature b) Rectification c) Volume d) none of these

5) $\int_S \vec{r}.d\vec{a} = -----$where V is the volume enclosed by the surface S.
a) V b) 3V c) $\frac{1}{3}V$ d) 2V

Total No. of Printed Pages:3

SUBJECT CODE NO:- B-2162
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. F.Y (Sem.-II) Examination OCT/NOV 2019
Mathematics MAT - 202
(Geometry)

[Time: 1:30 Minutes] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
i) Attempt all questions
ii) Figures to the right indicate full marks

Q.1 A) Attempt any one 08

a) Prove that every equation of the first degree in x,y,z represents a plane.

b) Find the equations of the line passing through a given point A(x,y,z) and having direction cosines l,m,n.

B) Attempt any one 07

c) Obtain the equation of the plane through the intersection of the planes $x + 2y + 3z + 4 = 0$ and $4x + 3y + 2z + 1 = 0$ and the origin

d) Prove that the lines
$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$;

$$\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$$

Intersect find also their point of intersection and the plane through them .

Q.2 A) Attempt any one 08

a) Find the length of the perpendicular from a given point P (x, y, z) to given line

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{Z-\gamma}{n}$$

b) Prove that the plane section of a sphere is a circle.

B) Attempt any one 07

c) Find the magnitude and the equations of the line of shortest distance between the lines

$$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$$

$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

d) Find the equation of the sphere through the circle

$$x^2 + y^2 + z^2 + 2x + 3y + 6 = 0$$
$$x - 2y + 4z - q = 0$$

And the centre of the sphere

$$x^2 + y^2 + z^2 - 2x + 4y - 6z + 5 = 0$$

Q.3 A) Attempt any one 05

a) Show that every section of a right circular cone by a plane perpendicular to its axis is a circle.

b) Find the points of intersection of the line

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{Z-\gamma}{n}$$

With the central conicoid

$$ax^2 + by^2 + cz^2 = 1$$

B) Attempt any one 05

c) Find the equation of the right circular cylinder of radius 2 whose axis is the line

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$$

d) Find the equations to the tangent planes to

$$7x^2 - 3y^2 - z^2 + 21 = 0$$

Which pass through the line

$$7x - 6y + 9 = 0\ , z = 3$$

Q.4 Choose the correct alternatives and fill the blanks 10

1) The equation to a plane in normal form is ------------

a) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

b) $\frac{x}{l} + \frac{y}{m} + \frac{z}{n} = 1$

c) $ax + by + cz = p$

d) $lx + my + nz = p$

2) Two lines which do not lie in the same plane are called -----------

a) Parallel b) intersecting c) coincident d) skew

3) The shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{4}$ is -----------

a) 1/6 b)($^{1}/\sqrt{6}$) c) ($\frac{1}{}\sqrt{3}$) d) 1/3

4) The radius of the sphere
$x^2 + y^2 + z^2 - 2x + 4y - 6z + 7 = 0$ is ---------
a) 49 b) 5 c) -7 d) $\sqrt{7}$

5) If a right circular cone has mutually perpendicular generators then semi – vertical angle is ----------
a) $tan^{-1}\sqrt{2}$ b) $tan^{-1} 2$ c) $\pi/4$ d) $\pi/2$

Total No. of Printed Pages:2

SUBJECT CODE NO:- B-2170
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc S.Y(Sem. -IV) Examination Oct/Nov 2019
Mathematics - MAT- 402
Partial Differential Equations

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B
1. All questions are compulsory.
2. Figures to the right indicate full marks.

Q.1 A) Attempt any one: 08

a) Obtain subsidiary equations for the Lagrange's linear partial differential equations.

b) Discuss the method for solving linear homogeneous partial differential equation with constant coefficients

$$F(D, D')z = f(x, y)$$

B) Attempt any one: 07

c) Solve
$x^2(y - z)p + (z - x)y^2q = z^2(x - y)]$

d) Find the complete integral of $\sqrt{p} + \sqrt{q} = 1$

Q.2 A) Attempt any one: 08

a) Explain Jacobi's method of solving partial differential equations.

b) Discuss Monge's method to solve

$$Rr + Ss + Tt = V$$

B) Attempt any one: 07

c) Solve
$$x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} = xy$$

d) Solve
$(D^2 + D'^2) = z = cosmx\ cosny$

Q.3 A) Attempt any one: 05

a) Discuss the method of general solution of the equation $F(D, D')z = e^{ax+by}$

b) Explain the method of solution of $Rr + Ss + Tt + f(x, y, z, p, q) = 0$ when $S^2 = 4RT$

B) Attempt any one: 05

c) Solve $\frac{\partial^2 z}{\partial x \partial y} = \frac{1}{xy}$

d) Solve $(D^2 - D' - 1)z = x^2 y$

Q.4 Choose the correct alternative and rewrite the sentence: 10

i) The general solution of the equation $(A_0 D^n + A_1 D^{n-1} D' + --- + A_n D'^A)z = 0$ is ----
   a) $z = \phi_1(y + m_1 x)$
   b) $z = \phi_1(y + m_1 x) + \phi_2(y + m_2 x)$
   c) $z = \phi_1(y + m_1 x) + \phi_2(y + m_2 x) + ---- + \phi_n(y + m_n x)$
   d) $z = \phi_1(y + x) + \phi_2(y + x) + ---- + \phi_n(y + x)$

ii) The auxiliary equations for the equations $xp + yq = z$ are ----------
   a) $dx = dy = dz$
   b) $\frac{dx}{p} = \frac{dy}{q} = \frac{dz}{z}$
   c) $dx = dy = \frac{dz}{z}$
   d) $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$

iii) The complete integral of the equation of the form $f_1(x, p) = f_2(y, q)$ is
   a) $z = \int \phi_1(x, c_1)dx + \int \phi_2(y, c_1)dy + b$
   b) $z = \int \phi_1(x, p)dx + \int \phi_2(y, q)dy + b$
   c) $z = \int \phi_1(x, c_1)dx + b$
   d) $z = \int \phi_2(y, c_1)dy + b$

iv) The solution of the equation $s = 2x + 2y$ is --------
   a) $z = x^2 + 2xy + \phi(y)$
   b) $z = x^2 y + xy^2 + f(f) + f(x)$
   c) $z = xy^2 + 2xy + f(x)$
   d) $z = 2xy + f(y)$

v) The function $z = e^{-\gamma x} \phi(\beta x - \alpha y)$ is a solution of equation -------
   a) $(D - mD' - k)z = 0$
   b) $(D + mD' + k)z = 0$
   c) $(\alpha D + \beta D' + \gamma)z = 0$
   d) $(\alpha D - \beta D' \gamma)z = 0$

Total No. of Printed Pages:2

SUBJECT CODE NO:- B-2169
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. S.Y (Sem.-IV) Examination OCT/NOV 2019
Mathematics MAT - 401
Numerical Methods

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
i) Attempt all questions
ii) Figures to the right indicate full marks
iii) Use of non-programmable calculator and logarithmic table is allowed

Q.1 A) Attempt any one: 08

a) Explain the method of false position for obtaining root of an equation f (x) = 0.
b) Derive Lagrange's interpolation formula.

B) Attempt any one: 07

c) Find a real root of the equation $f(x) = x^3 - x - 1 = 0$ by using Bisection method.
d) The population of a town in the decennial census was as given below. Estimate the population for the year 1895.

Year: x	1891	1901	1911	1921	1931
Population: y (In thousands)	46	66	81	93	101

Q.2 A) Attempt any one : 08

a) Prove that: $\mu \equiv \sqrt{1 + \frac{1}{4}\delta^2}$ with usual notations.
b) Explain the method of fitting the data points (xi, yi), i=1, 2,………..,m to a polynomial of the n^{th} degree.

B) Attempt any one: 07

c) Economize the power series

$$sinx \approx x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$$

d) Find the eigenvalues and eigenvectors of the matrix

$$A=\begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$$

Q.3 A) Attempt any one: 05

a) Describe the method to determine eigenvalues of symmetric tridiagonal matrix.
b) Explain the Euler's method to find solution of a differential equation $y' = f(x,y)$.

B) Attempt Any One: 05

c) Solve the system of equations
5x-2y+z=4
7x+y-5z=8
3x+7y+4z=10
By using Gaussian elimination method.

d) Solve the equation
$y' = x + y^2$, Subject to the condition y=1 when x=0 using picard's method of successive approximations.

Q.4 Choose the correct alternative and rewrite the sentence. 10

i) If a function f(x) is continuous between a and b then there exists at least one root of f(x)=0 between a and b if ________ .
a) f(a)>0 and f(b)>0
b) f(a)<0 and f(b)<0
c) f(a) and f(b) are of opposite signs.
d) f(a) f(b)=0

ii) $E^5 Y_2$= _____ , where E is shift operator
a) Y_7 b) Y_5 c) Y_3 d) Y_2

iii) The chebyshev polynomial of degree 2 is _______
a) $2x^2 + 1$ b) $2x^2 - 1$ c) $2x^2 + 2$ d) $2x^2 - 2$

iv) Eigenvalues of the matrix
$\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ are ________.

a) 1, 4 b) 2, 3 c) 1, 3 d) -1, -3

v) If $\frac{dy}{dx} = y - x, y(o) = 2$, then by second order Runge-kutta formula with h=0.1 K_1= ________.
a) 2 b) -2 c) -0.2 d) 0.2

Total No. of Printed Pages:3

SUBJECT CODE NO:- B-2162
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. F.Y (Sem.-II) Examination OCT/NOV 2019
Mathematics MAT - 202
(Geometry)

[Time: 1:30 Minutes] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
i) Attempt all questions
ii) Figures to the right indicate full marks

Q.1 A) Attempt any one 08

a) Prove that every equation of the first degree in x,y,z represents a plane.

b) Find the equations of the line passing through a given point A(x,y,z) and having direction cosines l,m,n.

B) Attempt any one 07

c) Obtain the equation of the plane through the intersection of the planes $x + 2y + 3z + 4 = 0$ and $4x + 3y + 2z + 1 = 0$ and the origin

d) Prove that the lines

$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$;

$$\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$$

Intersect find also their point of intersection and the plane through them .

Q.2 A) Attempt any one 08

a) Find the length of the perpendicular from a given point P (x, y, z) to given line

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{Z-\gamma}{n}$$

b) Prove that the plane section of a sphere is a circle.

B) Attempt any one 07

c) Find the magnitude and the equations of the line of shortest distance between the lines

$$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$$

$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

d) Find the equation of the sphere through the circle

$$x^2 + y^2 + z^2 + 2x + 3y + 6 = 0$$
$$x - 2y + 4z - q = 0$$

And the centre of the sphere

$$x^2 + y^2 + z^2 - 2x + 4y - 6z + 5 = 0$$

Q.3 A) Attempt any one 05

a) Show that every section of a right circular cone by a plane perpendicular to its axis is a circle.

b) Find the points of intersection of the line

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{Z-\gamma}{n}$$

With the central conicoid

$$ax^2 + by^2 + cz^2 = 1$$

B) Attempt any one 05

c) Find the equation of the right circular cylinder of radius 2 whose axis is the line

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$$

d) Find the equations to the tangent planes to

$$7x^2 - 3y^2 - z^2 + 21 = 0$$

Which pass through the line

$$7x - 6y + 9 = 0, z = 3$$

Q.4 Choose the correct alternatives and fill the blanks 10

1) The equation to a plane in normal form is ------------

a) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

b) $\frac{x}{l} + \frac{y}{m} + \frac{z}{n} = 1$

c) $ax + by + cz = p$

d) $lx + my + nz = p$

2) Two lines which do not lie in the same plane are called -----------

a) Parallel b) intersecting c) coincident d) skew

3) The shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{4}$ is -----------

a) 1/6 b)($^1/\sqrt{6}$) c) ($\frac{1}{}\sqrt{3}$) d) 1/3

4) The radius of the sphere
$x^2 + y^2 + z^2 - 2x + 4y - 6z + 7 = 0$ is ---------
a) 49 b) 5 c) -7 d) $\sqrt{7}$

5) If a right circular cone has mutually perpendicular generators then semi – vertical angle is ----------
a) $tan^{-1}\sqrt{2}$ b) $tan^{-1} 2$ c) $\pi/4$ d) $\pi/2$

Total No. of Printed Pages:2

SUBJECT CODE NO:- B-2169
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. S.Y (Sem.-IV) Examination OCT/NOV 2019
Mathematics MAT - 401
Numerical Methods

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
i) Attempt all questions
ii) Figures to the right indicate full marks
iii) Use of non-programmable calculator and logarithmic table is allowed

Q.1 A) Attempt any one: 08

a) Explain the method of false position for obtaining root of an equation f (x) = 0.
b) Derive Lagrange's interpolation formula.

B) Attempt any one: 07

c) Find a real root of the equation $f(x) = x^3 - x - 1 = 0$ by using Bisection method.
d) The population of a town in the decennial census was as given below. Estimate the population for the year 1895.

Year: x	1891	1901	1911	1921	1931
Population: y (In thousands)	46	66	81	93	101

Q.2 A) Attempt any one : 08

a) Prove that: $\mu \equiv \sqrt{1 + \frac{1}{4}\delta^2}$ with usual notations.
b) Explain the method of fitting the data points (xi, yi), i=1, 2,………..,m to a polynomial of the n^{th} degree.

B) Attempt any one: 07

c) Economize the power series
$sinx \approx x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$

d) Find the eigenvalues and eigenvectors of the matrix

$$A=\begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$$

Q.3 A) Attempt any one: 05

a) Describe the method to determine eigenvalues of symmetric tridiagonal matrix.
b) Explain the Euler's method to find solution of a differential equation $y' = f(x, y)$.

B) Attempt Any One: 05

c) Solve the system of equations
5x-2y+z=4
7x+y-5z=8
3x+7y+4z=10
By using Gaussian elimination method.

d) Solve the equation
$y' = x + y^2$, Subject to the condition y=1 when x=0 using picard's method of successive approximations.

Q.4 Choose the correct alternative and rewrite the sentence. 10

i) If a function f(x) is continuous between a and b then there exists at least one root of f(x)=0 between a and b if ________ .
a) f(a)>0 and f(b)>0
b) f(a)<0 and f(b)<0
c) f(a) and f(b) are of opposite signs.
d) f(a) f(b)=0

ii) $E^5 Y_2$= _____ , where E is shift operator
a) Y_7 b) Y_5 c) Y_3 d) Y_2

iii) The chebyshev polynomial of degree 2 is _______
a) $2x^2 + 1$ b) $2x^2 - 1$ c) $2x^2 + 2$ d) $2x^2 - 2$

iv) Eigenvalues of the matrix
$\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ are ________.

a) 1, 4 b) 2, 3 c) 1, 3 d) -1, -3

v) If $\frac{dy}{dx} = y - x, y(o) = 2$, then by second order Runge-kutta formula with h=0.1 K_1= ________.
a) 2 b) -2 c) -0.2 d) 0.2

Total No. of Printed Pages:02

SUBJECT CODE NO:- B-2165
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem.-VI) Examination OCT/NOV 2019
Mathematics MAT-601
Real Analysis-II

[Time: 1:30 Minutes] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
i. All questions are compulsory.
ii. Figures to the right indicate full marks.

Q.1 A) Attempt any one: 08

a) if (M, p) is a complete metric space for each $n \in I$, if F_n is a closed bounded subset of M such that $F_1 \supseteq F_2 \supseteq \cdots \supseteq F_n \supseteq F_{n+1} \supseteq \cdots$ and diam. $F_n \to 0$ as $n \to \infty$, then prove that $\bigcap_{n=1}^{\infty} F_n$ contains precisely one point.

b) If f is a continuous function from the compact metric space M_1 into a metric space M_2 , then prove that the range $f(M_1)$ of f is also compact.

B) Attempt any one: 07

c) If l^{∞} denote the set of all bounded sequences of real numbers, if $x = \{x_n\}_{n=1}^{\infty}$ and $y = \{y_n\}_{n=1}^{\infty}$ are the points in l^{∞}, define $\rho(x,y) = \underset{1 \le n < \infty}{\text{l.u.b}} |x_n - y_n|$, show that ρ is metric for l^{∞}.

d) If A and B are open subsets of R^1, then prove that $A \times B$ is a open subset of R^2.

Q.2 A) Attempt any one 08

a) If $f \in \Re[a,b]$ and $a < c < b$, then prove that $f \in \Re\,[a,c], f \in \Re[c,b]$ and also prove $\int_a^b f = \int_a^c f + \int_c^b f$

b) If the series $\frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$ converges uniformly to $f(x)$ on $[-\pi, \pi]$, then prove that it is the Fourier series for $f(x)$ on $[-\pi, \pi]$.

07

B) Attempt any one:

c) Show that: $\lim_{n\to\infty} \frac{1}{n}\left[\sin\frac{\pi}{n} + \sin\frac{2\pi}{n} + \cdots + \sin\frac{2\pi}{n}\right] = \frac{2}{\pi}$

d) Find the Fourier series expansion for $f(x) = |x|$ in $[-\pi, \pi]$.

Q.3 A) Attempt any one: 05

a) If A is subset of the metric space (m, ρ), and if (a, ρ) is compact , then prove that A is closed subset of (m, ρ).

b) If $f'(x) = 0$ for every x in the closed bounded interval [a,b], then prove that f is constant on [a,b].

B) Attempt any one: 05

c) If $f: R^2 \to R^2$ is defined by $f(x, y) = (y, x)$ for all $(x, y) \in R^2$, then show that f is continuous on R^2.

d) Prove that: $\frac{2\pi^2}{9} \leq \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{2x}{\sin x} dx \leq \frac{4\pi^2}{9}$.

Q.4 Choose the correct alternative and rewrite the sentence: 10

1) The discrete metric space (R, d) is denoted by ________.

a) R^d b) R_d c) R^1 d) l^∞

2) in a metric space M the subset E of M is closed if ________.

a) $E \subseteq \overline{E}$ b) $E = \phi$ c) $E = \overline{E}$ d) $E = R^1$

3) Union of infinite number of closed subsets of metric space_________.

a) Closed set b) Both open and closed set
c) Compact set d) need not be closed set

4) if f is continuous on [a,b], then threr exists $c \in (a, b)$such that $\int_a^b f(x)dx$ = __________.

a) $f(a)(b - c)$ b) $f(b)(c - a)$
c) $f(c)(b - a)$ d) $f(c)(b + a)$

5) For all n, $\int_{-\pi}^{\pi} \cos x dx$ =_________.

a) $-\pi$ b) π c) 0 d) 1

Total No. of Printed Pages:2

SUBJECT CODE NO:- B-2166
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem.-VI) Examination OCT/NOV 2019
Mathematics MAT - 602
Abstract Algebra – II

[Time: 1:30 Minutes] **[Max. Marks:50]**

Please check whether you have got the right question paper.

N.B
i. All questions are compulsory.
ii. Figures to the right indicate full marks.

Q.1 (A) Attempt any one: 08

(a) Prove that the kernel of a homomorphism T is a subspace of a vector space V, also that a homomorphism T is an isomorphism if and only if its kernel is (0).

(b) If $v_1, v_2, \dots, v_n$ are in a vector space V, then prove that either they are linearly independent or some v_k is linear combination of the preceding one $v_1, v_2, \dots, v_{k-1}$.

(B) Attempt any one: 07

(c) If V is finite-dimensional vector space and W is a subspace of V, then prove that there is a subspace $W_1$of V such that $V = W \oplus W_1$.

(d) If U is a vector space and W a subspace of U, then prove that there is a homo morphism of U onto U/W.

Q.2 (A) Attempt any one: 08

(a) If $V = F^{(n)}\ with\ (u,v) = \alpha_1\bar{\beta}_1 + \alpha_2\bar{\beta}_2 + \cdots + \alpha_n\bar{\beta}_n$ where $u = (\alpha_1, \alpha_2, \dots . \alpha_n)$ and $v = (\beta_1, \beta_2, \dots ., \beta_n)$, then prove that

$$\left|\alpha_1\bar{\beta}_1 + \alpha_2\bar{\beta}_2 + \cdots + \alpha_n\bar{\beta}_n\right|^2$$
$$\leq (|\alpha_1|^2 + |\alpha_2|^2 + \cdots + |\alpha_n|^2)(|\beta_1|^2 + |\beta_2|^2 + \cdots + |\beta_n|^2)$$

(b) If V is a finite-dimensional inner product space, then prove that V has an orthonormal set as basis.

(B) Attempt any one: 07

(c) In the vector space $F^{(n)}$ define for the vectors.

$$u = (\alpha_1, \alpha_2, \dots\ \alpha_n) \text{and } v = (\beta_1, \beta_2, \dots, \beta_n),$$
$$(u,v) = \alpha_1\bar{\beta}_1 + \alpha_2\bar{\beta}_2 + \cdots + \alpha_n\bar{\beta}_n,$$

then show that this defines as inner product on $F^{(n)}$.

(d) If F_2 is a family of polynomials of degree 2 at most. Define and inner product on $F_2 as$:

$$\big(p(x), q(x)\big) = \int_0^1 p(x)q(x)dx$$

If $\{1, x, x^2\}$ is a basis of the inner product space on F_2. Find an orthonormal basis from this basis.

Q.3 (A) Attempt any one 05

(a) If a, b, c are real numbers such tha $a > 0$ and $a\lambda^2 + 2b\lambda + c \geq 0$ for all real numbers $\lambda \geq 0$, then prove that $b^2 \leq ac$.

(b) If $v_1, v_2, \dots v_n \in V$ are linearly independent, then prove that every element in their span has a unique representation in the form $\lambda_1 v_1 + \lambda_2 v_2 + \cdots \dots + \lambda_n v_n$ with the $\lambda_i \in F$.

(B) Attempt any one: 05

(c) If A and B are submodules of M, then prove that

$$A + B = \{a + b | a \in A, b \in B\}$$

is a submodule of M.

(d) If V is finite-dimensional vector space and T is a homomorphism of V into itself which is not onto, then prove that there is some $v \neq 0$ in V such that $T(v) = 0$.

Q.4 Choose the correct alternative and rewrite the sentence 10

1. A vector space with inner product is called…………
   (a) dual space
   (b) second dual space
   (c) inner product space
   (d) annihilator

2. If V is an inner product space over F, then for $v \in V, \alpha \in F$, we have $\|\alpha u\| =$………..
   (a) $\alpha^2 \|u\|$
   (b) $\alpha \|u\|$
   (c) $\alpha \| \|u\|$
   (d) $|\alpha| \|u\|$

3. If W is a subspace of a vector space V over the field F, and if V/W is quotient space of W in V, then vector addition on V/W is defined as $(u + W) + (v + W) =$……………… for all $u, v \in V$.
   (a) $(u + v) + W$
   (b) $(u - v) + W$
   (c) $u - v$
   (d) $u + v$

4. If V is a vector space over a field F, then the elements of F are called…….
   (a) Scalars
   (b) Vectors
   (c) linearly independent vectors
   (d) linearly dependent vectors

5. The norm of the vector (1, 0, -1) is ……………
   (a) 0
   (b) -1
   (c) 1
   (d) $\sqrt{2}$

Total No. of Printed Pages:2

SUBJECT CODE NO:- B-2187
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem.-VI) Examination OCT/NOV 2019
Mathematics MAT
1) Mathematical Statistics-II – 603

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
1. All questions are compulsory
2. Figures to the right indicate full marks

Q.1 A) Attempt any one 08

1) Let $X_1, X_2, ------, X_n$ be n random variables then prove that:
$V[\sum_{i=1}^{n} a_i X_i] = \sum_{i=1}^{n} a_i^2 V(X_i) + 2\sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j Cov(X_i, X_j)$ i $< j$

2) If X and Y are independent random variables then prove that:
$$E\big(h(x).K(Y)\big) = E\big(h(x)\big).E\big(K(Y)\big).$$
where h(x)is a function of X alone
And K(Y) is a function of Y alone, provided expectations on both sides exist.

B) Attempt any one 07

3) Starting from the origin, unit steps are taken to the right with probability p and to the left with probability q (q=1-p) Assuming independent movements, find the mean and variance of the distance moved from origin after n steps.

Q.2 A) Attempt any one: 08

1) Find the first four moments of Binomial distribution by using recurrence relation.
2) Prove that Poisson distribution is a limiting case of Binomial distribution.

B) Attempt any one 07

3) If 'm' things are distributed among a men and b women . show that the probability that the numbers of things received by men is odd ,is
$$\frac{1}{2}\left[\frac{(b+a)^m - (b-a)^m}{(b+a)^m}\right]$$

4) In a book of 520 pages 390 typographical errors occur. Assuming poisson Law for the number of errors per page, find a probability that a sample of 5 pages will contain no error.

Q.3 A) Attempt any one 05

1) Define exponential distribution and hence find variance of exponential distribution
2) Find the median of normal distribution.

B) Attempt any one 05

3) If X has a Uniform distribution in [0,1], find the distribution (p.d.f) of – 2logX. Identify the distribution also.
4) If X and Y are independent Poisson variates with means $\lambda_1 and\ \lambda_2$ respectively, find the probability that $X + Y = K$

Q.4 Choose the correct alternative and rewrite the sentence. 10

1) Var $(2X \pm 3) = ------$
   a) 6 b) 5 c)-1 d) 4, if var(x)=1

2) Ten coins are thrown simultaneously then probability of getting exact 7 heads is --------
   a) 1/1024 b) 1/512 c) 15/128 d) 1/256

3) The first moment (about origin) of geometric mean is ---------
   a) q/p b) p/q c) 1/pq d) pq

4) Moment generating function of normal distribution is ----------
   a) $e^{\left(\mu t+\frac{t^2\sigma^2}{2}\right)}$ b) $e^{\left(\mu t-\frac{\sigma^2 t^2}{2}\right)}$ c) $e^{-\left(\mu t+\frac{t^2\sigma^2}{2}\right)}$ d) $e^{-\left(\mu t-\frac{t^2\sigma^2}{2}\right)}$

5) Moment generating function of rectangular distribution is ------------
   a) $\frac{e^{bt}+e^{at}}{t(b-a)}$ b) $\frac{e^{bt}-e^{at}}{t(b-a)}$ c) $\frac{e^{bt}-e^{at}}{t(b+a)}$ d) $\frac{e^{bt}+e^{at}}{t(b+a)}$

OR

Total No. of Printed Pages:02

SUBJECT CODE NO:- B-2187
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem.-VI) Examination OCT/NOV 2019
Mathematics MAT
2) Ordinary Differential Equation-II - 604

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Attempt any one:- 08

a) Let $\Phi_1, \Phi_2, \dots\dots, \Phi_n$ be the n solutions of
$L[y] = y^{(n)} + a_1(x)y^{(n-1)} + - - - - - + a_n(x)y = 0$ on I satisfying
$$\Phi_i^{(i-1)}(x_0) = 1, \Phi_i^{(j-1)}(x_0) = 0, \quad j \neq i$$
If Φ is any solution of L[y]=0 on I then prove that there exists n constants $C_1, C_2, \dots\dots C_n$ such that $\Phi = c_1\Phi_1 + c_2\Phi_2 + - - - - c_n\Phi_n$.

b) If Φ_1 is a solution of L[y]=y"+a_1 (x)y' +a_2(x)y = 0 on an interval I and $\Phi_1(x) \neq 0$ On I , a second solution Φ_2 of L[y]=0 on I is given by
$\Phi_2(x) = \Phi_1(x) \int_{x_0}^{x} \frac{1}{[\Phi_1(s)]^2} \exp\left[-\int_{x_0}^{x} a_1(t)dt\right] ds$
Prove that the functions Φ_1, Φ_2 form a basis for the solutions of $L[y] = 0$ on I.

B) Attempt any one:- 07

c) Show that $\Phi(x) = x^r$ r is constant is a solution of the equation
$y'' + \frac{1}{x}y' - \frac{1}{x^2}y = 0 \quad for\ x > 0$. Find two linearly independent solutions for x>0 and prove that they are linearly independent.

d) Show that $\Phi_1(x) = x$, $(0 < x < 1)$ is a solution of
$(1 - x^2)y'' - 2xy' + 2y = 0$ and find a second independent solution.

Q.2 A) Attempt any one:- 08

a) Prove that, with usual notations
$\Psi_p(x) = \sum_{K=1}^{n} \Phi_K(x) \int_{x_0}^{x} \frac{W_K(t)b(t)}{W(t)}\, dt$ is a particular solution of
$L[y] = y^{(n)} + a_1(x)y^{(n-1)} + - - - + a_n(x)y = b(x)$

b) Obtain the Bessel function of zero order of the first kind. Does it converge? Justify.

B) Attempt any one: 07

c) Show that

$\int_{-1}^{1} P_n(x) P_m(x) dx = 0\,,\ n \neq m$

d) Find all solutions of the equation
$2x^2 y'' + xy' - y = 0$ for x>0

Q.3 A) Attempt any one:- 05

a) Prove that the function
$\Phi_2(x) = \sum_{k=0}^{\infty} c_k x^k + (\log x)\Phi_1(x), \quad (c_0 = 0)$
where $\Phi_1(x) = J_0(x)$ is a solution of the Bessel equation of order α.

b) Prove that a basis for solution of the Euler equation on any interval not containing x=0 is $\Phi_1(x) = |x|^{r_1}$, $\Phi_2(x) = |x|^{r_2}$ if $r_1 \neq r_2$

B) Attempt any one:- 05

c) Show that $J_{\alpha-1}(x) - J_{\alpha+1}(x) = 2J_{\alpha}{}'(x)$

d) Find all solutions of
$3x^2 y'' + 5xy' + 3xy = 0$

Q.4 Choose the correct alternative and rewrite the sentence:- 10

1) The equation $(x-1)^2 y'' + 2xy' - 8y = 0$ has a singular point.
a) 0 b) 1 c) -1 d) 2

2) The value of the integral $\int_{-1}^{1} P_S^2(x) dx$ is
a) $\frac{2}{11}$ b) $\frac{11}{2}$ c) $\frac{2}{5}$ d) $\frac{5}{2}$

3) The value of the Wronskian of $\Phi_1(x) = x^2, \Phi_2(x) = x^3, x \neq 0$ is
a) $3x^2$ b) x^2 c) x^4 d) $4x^3$

4) The number of linearly independent solutions of the equation.
$x^3 y'' + x^2$ y $+ 3xy' + 4y = 0$ is
a) 1 b) 2 c) 3 d) 4

5) If $W' + a_1(x)W = 0$ then W(x) is given by
a) $W(x) = 0$ b) $W(x) \neq 0$ c) $W(x) = e^{cx}$ d) $W(x) = ce^{-\int_{x_0}^{x} a_1(t)dt}$

OR

Total No. of Printed Pages:01

SUBJECT CODE NO:- B-2187
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem.-VI) Examination OCT/NOV 2019
Mathematics MAT
3) Programming in C-II-605

[Time: 1:30 Hours] **[Max.Marks:40]**

Please check whether you have got the right question paper.

N.B
1. All questions are compulsory.
2. Assume the data whenever not given with justification
3. Figures to the right indicate full marks.

Q.1 A) Attempt any one: 05
- a) Explain how decision making is done using if statement.
- b) State dangling else problem? How to resolve it?

B) Attempt any one 05
- c) Write a program to determine range of values and average cost of a personal computer.
- d) Explain rules for switch statement.

Q.2 A) Attempt any one 05
- a) Explain do statement in detail with example.
- b) Discuss how to skip a part of the body of the loop under certain conditions.

B) Attempt any one 05
- c) Write a program to evaluate
  $y = x^n$
  Where n is a non-negative integer.
- d) Write a program to calculate the sum of squares of all integers between 1 and 15.

Q.3 A) Attempt any one 05
- a) Write a short note on data structures.
- b) Discuss in detail searching and sorting.

B) Attempt any one 05
- c) Write a C program to evaluate standard deviation of given data (Assume the data)
- d) Write a program for initializing large arrays when runtime is at 1.0.

Q.4 Fill in the blanks: 10
- a) If the test expression is true, the statement –block will be ________ executed.
- b) A counter- controlled loop is called _________.
- c) The general form of array declaration is _____.
- d) The __________ operator is a combination of ? and _______
- e) Switch is a multiway ______ statement.

Total No. of Printed Pages:2

SUBJECT CODE NO:- B-2188
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. S.Y (Sem-IV) Examination OCT/NOV 2019
Mathematics MAT – 403
Mechanics-II

[Time: 1:30 Minutes] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B i) All questions are compulsory.
ii) Figures to the right indicate full marks.
iii) Draw well labelled diagrams wherever necessary.

Q.1 (A) Attempt any one: 08

(a) If the sum of external forces acting on a system of particles is zero in any direction, then prove that the total momentum of the system in that direction remains same during the motion.

(b) Prove that the necessary and sufficient condition for a force to be conservative force is that line integral over a closed Path C in a conservative field is zero.

(B) Attempt any one: 07

(c) A point moves in a curve so that its tangential and normal accelerations are equal and the tangent rotates with uniform angular velocity. Show that intrinsic equation of path is of the form $s = Ae^{\psi} + B$.

(d) A particle of mass 0.1 lb. Has the velocity $\overrightarrow{2i} + \overrightarrow{3j}$ fit/ sec at $t = 2\ sec$. It is subjected to a force $3t^2\vec{i} + \cos(\pi t)\,\vec{j}$. Find the impulse of the force over the interval $2 \le t \le 3$. Also find the velocity at $t = 3\ sec$.

Q.2 (A) Attempt any one: 08

(a) A shell bursts on striking a ground and its pieces fly in all directions, with maximum speed v. Find the time for which a person at a distance a is in danger.

(b) Find the differential equation of the path of a particle moving under a central force $f(r)$ directed towards a fixed point O in a plane in polar form.

(B) Attempt any one: 07

(c) Two particles are projected from the same point in the same verticle plane with equal velocities. If t, t' be the times taken to reach other common point of their paths and T and T' be the times to the highest points, then show that $(tT + t'T')$ is independent of the directions of projection.

(d) Find the law of the central force under which the curve $r^n = a^n \cos n\theta$ can be described.

Q.3 (A) Attempt any one: 05

(a) Find the Cartesian equation of the path of the projectile.

(b) Find the components of velocity and acceleration along rectangular Cartesian axes.

(B) Attempt any one: 05

(c) Find the work done by the force $\vec{F} = 2x\vec{i} + 2y\vec{j}$ in a moving particle from $P(-1,0)$ to $Q(4,2)$.

(d) A shell bursts on contact with the ground and pieces from it fly in all directions with velocities up to 80 ft/sec. Show that the man 100 ft away is in danger for $\frac{5\sqrt{2}}{2}$ seconds.

Q.4 Choose the correct alternative and rewrite the sentence: 10

(1) The magnitude of the velocity is called ………………
- (a) Displacement
- (b) Speed
- (c) Acceleration
- (d) angular acceleration

(2) Unit of angular acceleration is ……….
- (a) cm/sec^2
- (b) rad/sec
- (c) $ft/\sec^2$
- (d) $rad/\sec^2$

(3) Actions and reactions are …………
- (a) not equal not opposite
- (b) not equal but opposite
- (c) equal and opposite
- (d) equal but not opposite

(4) The time taken by the particle to come back to the horizontal plane through the point of projection is called……………..
- (a) Projectile
- (b) the time of flight
- (c) horizontal range
- (d) impulse

(5) If the force is acting away from a fixed point then it is called …………
- (a) Central repulsive force
- (b) tangential force
- (c) terminal force
- (d) central attractive force

Total No. of Printed Pages:02

SUBJECT CODE NO:- B-2026
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y. (Sem-V) Examination Oct/Nov 2019
Mathematics MAT - 502
Abstract Algebra - I

[Time: 1:30 Hours] **[Max. Marks:50]**

Please check whether you have got the right question paper.

N.B
1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Attempt any one: 08

a) If H and K are subgroups of G, then prove that HK is a subgroup of group G if and only if HK=KH.

b) If ∅ is a homomorphism of G onto $\bar{G}$ with kernel K, then prove that K is normal subgroup of G.

B) Attempt any one: 07

c) If G is the group of all complex numbers a+ib, a, b are real, not both zero, under multiplication, and if $H = \{a + ib | a^2 + b^2 = 1\}$, then show that H is a subgroup of G.

d) Show that the intersection of two normal subgroups of G is also normal subgroup of G.

Q.2 A) Attempt any one: 08

a) Prove that the homomorphism ∅ of a ring R into a ring R' is an isomorphism if and only if $I(\emptyset) = (0)$, where $I(\emptyset)$ denotes the kernel of ∅

b) If $f(x), g(x)$ are two non-zero elements of the polynomial ring F[x], then prove that $\deg f(x) \cdot g(x) = deg f(x) + deg g(x)$

B) Attempt any one: 07

c) If R is a ring with unit element 1 and ∅ is homomorphism of R onto R', then prove that $\emptyset(1)$ is unit element of R'.

d) If R is the ring of all real valued continuous functions on interval [0,1] and if $M = \{f(x) \in R | f(\gamma) = 0 \; where \; 0 \leq \gamma \leq 1\}$, then prove that M is maximal ideal of R.

Q.3 A) Attempt any one:- 05

a) If G is a group then prove that the identity element of G is unique.

b) If p is prime number then prove that J_p, the ring of integers mod p is a field.

B) Attempt any one:- 05

c) If G is the group of integers under addition, H the subset consisting of all multiples of n, then show that H is subgroup of G.

d) If R and R' are any two arbitrary rings, where $R = R'$ and define $\emptyset: R \rightarrow R'$ by $\emptyset(a) = a$ for all $a \in R$ then show that $\emptyset$ is homomorphism. Also find the kernel of $\emptyset$.

Q.4 Choose the correct alternative and rewrite the sentence: 10

1) If every element of the group G is its own inverse then the group G is -----------
   a) Quotient group
   b) Normal subgroup
   c) Abelian group
   d) Non-abelian group

2) If $G = \{\pm 1, \pm i, \pm j, \pm k\}$ is a group of quaternions then $o(G) = -----$
   a) 0
   b) 2
   c) 4
   d) 8

3) If H is a subgroup of a group G, and if a, b ∈ G, then ----------
   a) $aH \neq bH \text{ and } aH \cap bH = \emptyset$
   b) $aH = bH \text{ or } aH \cap bH \neq \emptyset$
   c) $aH = bH \text{ or } aH \cap bH = \emptyset$
   d) $aH \neq bH \text{ and } aH \cap bH \neq \emptyset$

4) If $(R, +, \cdot)$ is a ring, then $(R, +)$ is ----------
   a) group
   b) Abelian group
   c) Commutator group
   d) finite group

5) Zero element of the quotient ring R/U is ----------
   a) R
   b) $R + U$
   c) $U + 1$
   d) U

B-2021

Total No. of Printed Pages:02

SUBJECT CODE NO:- B-2021
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. F.Y. (Sem-I) Examination Oct/Nov 2019
Mathematics MAT - 101
Differential Calculus

[Time: 01:30 Hours] **[Max. Marks:50]**

Please check whether you have got the right question paper.

N.B
1) Attempt all questions.
2) Figures to the right indicate full marks.

Q.1 A) Attempt any one: 08

a) If $y = cosech^{-1}x$, then find $\frac{dy}{dx}$.

b) If u and v be two functions of x possessing derivatives of the n[th] order, then prove that $(uv)_n = u_n v + n_{C_1} u_{n-1} v_1 + n_{C_2} u_{n-2} v_2 + ----+ n_{C_r} u_{n-r} v_r + ---+ n_{C_n} u\, v_n$

B) Attempt any one: 07

c) If $y = e^{ax} \cos^2 x \;\; sinx$, then find $\frac{d^n y}{dx^n}$.

d) If $y = x^2 \sin x$, prove that

$$\frac{d^n y}{dx^n} = (x^2 - n^2 + n)\sin\left(x + \frac{n\pi}{2}\right) - 2nx\cos\left(x + \frac{n\pi}{2}\right)$$

Q.2 A) Attempt any one: 08

a) If two functions f(x) and F(x) are derivable in a closed interval [a, b] and $F'(x) \neq 0$ for any value of x in [a,b] then prove that there exists at least one value 'c' of x belonging to the open interval (a, b) such that

$\frac{f(b)-f(a)}{F(b)-F(a)} = \frac{f'(c)}{F'(c)}$

b) If z = f(x,y) be a homogeneous function of x, y of degree n, then prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz, \forall\, x, y\, \epsilon$ the domain of the function.

B) Attempt any one: 07

c) Verify Rolle's theorem for the function $f(x) = (x-a)^m (x-b)^n$; m, n being positive integer , $x\, \epsilon [a, b]$

d) If $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$; $x^2 + y^2 + z^2 \neq 0$,

Show that

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

Q.3 A) Attempt any one:- 05

a) Prove that div $\vec{f}$ and curl $\vec{f}$ are point functions.

b) Prove that

$$curl\left(\phi\overrightarrow{f}\right) = grad\ \phi x\ \overrightarrow{f} + \phi\ curl\ \overrightarrow{f}$$

B) Attempt any one:- 05

c) Find grad ϕ if $\phi = 2x^2y^3 - 3y^2z^3$ at the point (1,-1,1)

d) If f is finitely derivable at c, then prove that f is also continuous at c.

Q.4 Choose the correct alternative: 10

i) For $x \in R$, cosh(-x) = ………

a) coshx b) –coshx c) sinhx d) –sinhx

ii) If y= sin(3x+5), then $y_3 = ----$

a) $3^2 \sin(3x + 5 + 3\frac{\pi}{2})$

b) $3^3 \sin(3x + 5 + 3\frac{\pi}{2})$

c) $3^3 \cos(3x + 5 + 3\frac{\pi}{2})$

d) None of these

iii) For $\forall\ x \in R,\ e^x = ------$

a) $1 + x + x^2 + ----$

b) $1 - x + x^2 - \ ----$

c) $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + ----$

d) $1 + x + \frac{x^2}{2!} + ----$

iv) grad $(\vec{r}.\vec{a})$= ---------

a) 0 b) $\vec{a}$ c) $2\vec{a}$ d) $3\vec{a}$

v) If ϕ is constant then grad ϕ = --------

a) 0 b) 2 c) 1 d) -1

Total No. of Printed Pages:03

SUBJECT CODE NO:- B-2022
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. F.Y. (Sem-I) Examination Oct/Nov 2019
Mathematics MAT - 102
Differential Equations

[Time: 01:30 Hours] **[Max. Marks:50]**

Please check whether you have got the right question paper.

N.B
1) Attempt all questions.
2) Figures to the right indicates full marks.

Q.1 A) Attempt any one:- 08

a) Prove that the necessary and sufficient condition of the differential equation $Mdx + Ndy = 0$, being exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

b) Explain the method of solving the differential equation $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x or constants.

B) Attempt any one:- 07

c) Solve the simultaneous equations:

$$\frac{dx}{dt} - 7x + y = 0$$

$$\frac{dy}{dt} - 2x - 5y = 0$$

d) Solve $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = \cos 2x$

Q.2 A) Attempt Any One:- 08

a) Explain the method of solving the differential equation

$$\frac{d^ny}{dx^n} + P_1\frac{d^{n-1}y}{dx^{n-1}} + ------ + P_n \cdot y = X,$$

Where $P_1, P_2,$ ------------ P_n are constants and X is a function of x.

b) Explain the method of solving the differential equation.

$$(a+bx)^n\frac{d^ny}{dx^n} + P_1(a+bx)^{n-1}\frac{d^{n-1}y}{dx^{n-1}} + ---- + P_{n-1}(a+bx)\frac{dy}{dx} + P_ny = f(x)$$

where $P_1, P_2, ------ P_n$ are constants.

B) Attempt any one:- 07

c) Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$

d) Solve $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$

Q.3 A) Attempt any one:- 05

a) With usual notation, prove that
$\frac{1}{f(D)}e^{ax}.V = e^{ax}.\frac{1}{f(D+a)}.V$
where V be any function of x.

b) Explain the method of solving the equation of the form

$$\frac{d^ny}{dx^n} = f(x)$$

B) Attempt any one:- 05

c) Solve $(a^2 - 2xy - y^2)dx - (x+y)^2dy = 0$

d) Form the partial differential equation by eliminating arbitrary constants a and b from the equation.

$$z = ax + by + ab$$

Q.4 Choose the correct alternative: 10

i) The integrating factor of the differential equation $x\frac{dy}{dx} - ay = x + 1$ is ---------
a) $\frac{1}{x^a}$ b) x^a c) $\frac{1}{x}$ d) $\frac{1}{a}$

ii) The general solution of the equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 54y = 0$ is ----------
a) $y = C_1e^{6x} + C_2e^{-9x}$
b) $y = C_1e^{-6x} + C_2e^{3x}$
c) $y = C_1e^{6x} + C_2e^{4x}$
d) None of the above

iii) The particular integral of the equation $x^2\frac{d^2y}{dx^2} + 7x\frac{dy}{dx} + 5y = x^5$ is --------
a) $\frac{x^5}{60}$ b) x^5 c) $\frac{x^5}{30}$ d) $\frac{x^5}{6}$

iv) The partial differential equation corresponding to $z = ax + by + ab$ is -------------

a) z=px+qy
b) z=pq
c) z=px+qy+pq
d) None of the above

v) The ordinary differential equation and partial differential equation are differ by

a) Their number of independent variable
b) Their number of dependent variable
c) Their total derivative
d) None of the above

Total No. of Printed Pages:2

SUBJECT CODE NO:- B-2029
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. S.Y. (Sem-III) Examination Oct/Nov 2019
Mathematics MAT – 301
Number Theory

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

i) Attempt all questions.
ii) Figures to the right indicate full marks.

Q.1 (A) Attempt any one 08

a) Prove that the linear congruence $ax \equiv b(mod\ n)$ has a solution if and only if d/b where $d = \gcd(a, n)$ If d/b, then show that it has d mutually incongruent solutions modulo n.
b) If p and q are distinct primes with $a^p \equiv a(mod\ q)$and $a^q \equiv a(mod\ p)$then prove that $a^{pq} \equiv a(mod\ pq)$.

(B) Attempt any one 07

c) Solve the set of simultaneous congruences
$$x \equiv 1(mod\ 3), x \equiv 2(mod\ 5), x \equiv 3(mod\ 7)$$

d) Find the remainder when 15! is divided by 17

Q.2 (A) Attempt any one 08

a) Show that τ and σ are both multiplicative functions.
b) If the integer n>1 has the prime factorization $n = P_1^{K_1} P_2^{K2} \ldots\ldots\ldots P_r^{Kr}$ then prove
$$\emptyset(n) = n\left(1 - \frac{1}{P_1}\right)\left(1 - \frac{1}{P_2}\right)\ldots.\left(1 - \frac{1}{P_r}\right)$$

(B) Attempt any one 07

c) Determine all solutions in positive integers for Diophantine equation $172x + 20y = 1000$.
d) Show that $18! \equiv -1(mod\ 437)$

Q.3 (A) Attempt any one 05

a) If $a = qb + r$ then prove that $\gcd(a, b) = \gcd(b, r)$
b) If P is prime and P/ab, then prove that P/a or P/b

(B) Attempt any one 05

c) By using Euler's theorem show that for any integer a,

$$a^{37} \equiv a(mod\ 1729).$$

d) Calculate $\phi(5040)$

Q.4 Choose the correct alternatives 10

i) gcd (12378, 3054) is --------

a) 6 b) 4 c) 7 d) 8

ii) If for any two positive integers 3054 and 12378, gcd(3054, 12378)=6 then lcm (3054, 12378) is------------

(a) 630402 (b) 6400402

(c) 6300402 (d) 6500402

iii) If ca=cb(mod n), then $a \equiv b\left(\text{mod}\,\frac{n}{d}\right)$ is true only if ---------

(a) d=gcd(a,n) (b) d = gcd(c,n)

(c) d=gcd(b,n) (d) d = gcd(a,b)

iv) Value of $\sigma(12)$ is --------------

a) 16 b) 27 c) 15 d) 28

v) Value of $\sum_{n=1}^{6} \tau(n)$ is --------------

a) 14 b) 12 c) 10 d) 8

Total No. of Printed Pages:2

SUBJECT CODE NO:- B-2030
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. S.Y. (Sem-III) Examination Oct/Nov 2019
Mathematics MAT - 302
Integral Transforms

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

i) All questions are compulsory.

ii) Figures to the right indicate full marks.

Q.1 (A) Attempt any one: 08

a) Prove that, $\beta(l,m) = \frac{\Gamma(l)\Gamma(m)}{\Gamma(lpm)}$, where ml are positive integer.

b) Prove that, $2^n\Gamma\left(n+\frac{1}{2}\right) = 1.3.5 \ldots\ldots\ldots\ldots(2n-1).\sqrt{\pi}$, where n is positive integer.

(B) Attempt any one: 07

c) Evaluate $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+x)^{m+n}}dx$

d) Find $\mathrm{L}\{e^{-t}(3\sinh 2t - 5\cosh 2t)\}$

Q.2 (A) Attempt any one: 08

a) Derive relationship between Fourier transform and Laplace transform.

b) Prove that, $\mathrm{L}^{-1}\left\{\frac{1}{(S^2+a^2)^2}\right\} = \frac{1}{2a^3}\{\mathrm{sinat} - \mathrm{atcosat}\}$

(B) Attempt any one: 07

c) Solve by using Laplace transform

$$(D^2+25)y = 6\cos(3t), with\ y(0) = 2, y'(0) = 0$$

d) Find $L^{-1}\left\{\tan^{-1}\frac{8}{S^2}\right\}$

Q.3 (A) Attempt any one: 05

a) If $L\{F(t)\} = f(s)$, then prove that

$$L\{t^nF(t)\} = (-1)^n\frac{d^nf(s)}{ds^n}$$

b) If $F\{F(x)\} = f(s)$, then prove that $F\{F(x-a)\} = e^{-isa}f(s)$.

(B) Attempt any one: 05

c) Find the Fourier transform of $f(x) = \frac{1}{x}$.

d) Find $L^{-1}\left\{\frac{S^2}{(S^2-4)^2}\right\}$

Q.4 For each of the following questions four alternatives are given for the answer. Only one of them is correct. Choose the correct alternative. 10

1) The value of $\Gamma(1)\,\Gamma(2)\,\Gamma(3)\ldots.\Gamma(9)$is…….

a) $\frac{2\pi}{\sqrt{10}}$

b) $\frac{(2\pi)^9}{\sqrt{10}}$

c) $\frac{(2\pi)^{\frac{9}{2}}}{\sqrt{10}}$

d) $\frac{\pi^{\frac{9}{2}}}{\sqrt{10}}$

2) The value of $\Gamma\left(\frac{1}{2}\right)$is ……………
   a) 0
   b) π
   c) $\sqrt{\pi}$
   d) π^2

3) The value of $L\{\cosh(at)\}$ is………………
   a) $\frac{a}{S^2-a^2}$
   b) $\frac{S}{S^2-a^2}$
   c) $\frac{S}{S^2+a^2}$
   d) $\frac{a}{S^2+a^2}$

4) The value of $F\{e^{ax}\}$ is …………..
   a) $\frac{S}{S^2+a^2}$
   b) $\frac{a}{S^2+a^2}$
   c) $\frac{S}{S^2-a^2}$
   d) None of these

5) The value of $L\{t^3\}$ is ………
   a) $\frac{6}{S^4}$
   b) $\frac{1}{S^4}$
   c) S^4
   d) $\frac{S^4}{6}$

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SUBJECT CODE NO:- B-2025
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y. (Sem-V) Examination Oct/Nov 2019
Mathematics MAT - 501
Real Analysis – I

[Time: 1.30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
1) All questions are compulsory
2) Figures to the right indicate full marks

Q.1 a) Attempt any one

i) Prove that the set of all rational numbers is countable. 08

ii) If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent to L, then prove that $\{S_n\}_{n=1}^{\infty}$ cannot also converge to a limit distinct from L. 08

b) Attempt any one :

iii) If B is an infinite subset of the countable set A, then prove that B is countable. 07

iv) Show that the sequence $\{\log(1/n)\}_{n=1}^{\infty}$ diverges to minus infinity 07

Q.2 a) Attempt any one

i) If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ converges, then prove that $\{S_n\}_{n=1}^{\infty}$is a Cauchy sequence. 08

ii) Prove that any bounded sequence of real numbers has a convergent subsequence 08

b) Attempt any one

iii) If $x = r\cos\theta,\ \ y = r\,sin\theta$ find $\frac{\partial(x,y)}{\partial(r,\theta)}$ 07

iv) Prove that 07

$$\frac{\partial(y_1, y_2, - - - - - -, y_n)}{\partial(x_1, x_2, - - - - - -, x_n)} \cdot \frac{\partial(x_1, x_2, - - - - - -, x_n)}{\partial(y_1, y_2, - - - - - -, y_n)} = 1$$

Q.3 a) Attempt any one

i) If $\sum_{n=1}^{\infty} a_n$ converges absolutely , then prove that $\sum_{n=1}^{\infty} a_n$ converges 05

ii) If $\sum_{n=1}^{\infty} a_n$ converges to A and $\sum_{n=1}^{\infty} b_n$ converges to B, then prove that 05

$\sum_{n=1}^{\infty}(a_n + b_n)$ converges to A+B

b) Attempt any one

iii) Find the values of x for which the series $\sum_{n=1}^{\infty} x^n/n$ converges absolutely 05

iv) Prove that the series 05
$\sum_{n=1}^{\infty}[1/n\ (n+1)]$ converges

Q.4 Choose the correct alternative and rewrite the sentence. 10

i) If X_A is the characteristic function of set A then $X_{AUB} = -------$

a) $X_A + X_B$
b) $X_A + X_B + X_{A\cap B}$
c) $X_A + X_B - X_{A\cap B}$
d) $X_A - X_B + X_{A\cap B}$

ii) The sequence $\left\{\sqrt{n}\right\}_{n=1}^{\infty}$ --------

a) Diverges to minus infinity b) converges to o
c) converges to n d) diverges to infinity

iii) If $\{s_n\}_{n=1}^{\infty}$ $where\ S_n = (-1)^n\ then\ \lim_{n\to\infty} inf\ S_n = -----$

a) 1 b) o c) ∞ d) -1

iv) If $\sum_{n=1}^{\infty} a_n$ is a series of real numbers and if $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty}|a_n|$ diverges then $\sum_{n=1}^{\infty} a_n$----------

a) Converges conditionally b) Converges c) oscillate d) diverges to ∞

v) If $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} b_n$diverges then $\sum_{n=1}^{\infty}(a_n + b_n) ----$

a) Converges absolutely b) converges conditionally c) Oscillate d) diverges

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SUBJECT CODE NO:- B-2026
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y. (Sem-V) Examination Oct/Nov 2019
Mathematics MAT - 502
Abstract Algebra - I

[Time: 1:30 Hours] **[Max. Marks:50]**

Please check whether you have got the right question paper.

N.B
1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Attempt any one: 08

a) If H and K are subgroups of G, then prove that HK is a subgroup of group G if and only if HK=KH.

b) If $\emptyset$ is a homomorphism of G onto $\bar{G}$ with kernel K, then prove that K is normal subgroup of G.

B) Attempt any one: 07

c) If G is the group of all complex numbers a+ib, a, b are real, not both zero, under multiplication, and if $H = \{a + ib | a^2 + b^2 = 1\}$, then show that H is a subgroup of G.

d) Show that the intersection of two normal subgroups of G is also normal subgroup of G.

Q.2 A) Attempt any one: 08

a) Prove that the homomorphism $\emptyset$ of a ring R into a ring R' is an isomorphism if and only if $I(\emptyset) = (0)$, where $I(\emptyset)$ denotes the kernel of $\emptyset$

b) If $f(x), g(x)$ are two non-zero elements of the polynomial ring F[x], then prove that $\deg f(x) \cdot g(x) = deg f(x) + deg g(x)$

B) Attempt any one: 07

c) If R is a ring with unit element 1 and $\emptyset$ is homomorphism of R onto R', then prove that $\emptyset(1)$ is unit element of R'.

d) If R is the ring of all real valued continuous functions on interval [0,1] and if $M = \{f(x) \in R | f(\gamma) = 0 \ where \ 0 \leq \gamma \leq 1\}$, then prove that M is maximal ideal of R.

Q.3 A) Attempt any one:- 05

a) If G is a group then prove that the identity element of G is unique.

b) If p is prime number then prove that J_p, the ring of integers mod p is a field.

B) Attempt any one:- 05

c) If G is the group of integers under addition, H the subset consisting of all multiples of n, then show that H is subgroup of G.

d) If R and R' are any two arbitrary rings, where $R = R'$ and define $\emptyset : R \to R'$ by $\emptyset(a) = a$ for all $a \in R$ then show that $\emptyset$ is homomorphism. Also find the kernel of $\emptyset$.

Q.4 Choose the correct alternative and rewrite the sentence: 10

1) If every element of the group G is its own inverse then the group G is -----------
   a) Quotient group
   b) Normal subgroup
   c) Abelian group
   d) Non-abelian group

2) If $G = \{\pm 1, \pm i, \pm j, \pm k\}$ is a group of quaternions then $o(G) = -----$
   a) 0
   b) 2
   c) 4
   d) 8

3) If H is a subgroup of a group G, and if a, b ∈ G, then ----------
   a) $aH \neq bH \text{ and } aH \cap bH = \emptyset$
   b) $aH = bH \text{ or } aH \cap bH \neq \emptyset$
   c) $aH = bH \text{ or } aH \cap bH = \emptyset$
   d) $aH \neq bH \text{ and } aH \cap bH \neq \emptyset$

4) If $(R, +, \cdot)$ is a ring, then $(R, +)$ is ----------
   a) group
   b) Abelian group
   c) Commutator group
   d) finite group

5) Zero element of the quotient ring R/U is ----------
   a) R
   b) $R + U$
   c) $U + 1$
   d) U

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SUBJECT CODE NO:- B-2048
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. S.Y. (Sem-III) Examination Oct/Nov 2019
Mathematics MAT - 303
Mechanics-I

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B
1) Attempt all questions.
2) Figures to the right indicate full marks.
3) Draw well-labelled diagrams whenever necessary.

Q.1 A) Attempt any one 08

a) Find the resultant of two unlike parallel forces acting upon a rigid body.

b) Prove that if the three forces acting on a particle be represented in magnitude and direction by the three sides of a triangle, taken in order, then the forces are in equilibrium.

B) Attempt any one 07

c) If the resultant R divides the angle between the two forces P and Q in the ratio 1:2. Prove that $R = \frac{P^2 - Q^2}{Q}$

d) A body of weight 52 kg is suspended by two strings of length 5m and 12m attached to the points in the same horizontal line whose distance apart is 13m. Find the tensions of the strings.

Q.2 A) Attempt any one 08

a) Prove that the sum of the vector moments of two like parallel forces acting on a rigid body about any point equals to the vector moment of their resultant about the same point.

b) Determine the magnitude and direction of the resultant $\vec{R}$ of two forces $\vec{P}$ and $\vec{Q}$ acting at an angle θ

B) Attempt any one 07

c) Prove that the vector moment of the resultant couple of two couples acting upon a rigid body is the sum of the vector moments of the given couples.

d) A small heavy ring of weight W is free to slide on a smooth vertical circular wire of radius a. one extremity of a light inextensible string of length l is attached to the ring while the other to the highest point of the wire. Find the tension of the string and the reaction of the wire.

Q.3 A) Attempt any one 05

a) Prove that the sum of the vector moments of a system of forces acting on a particle about any point equals to the vector moment of their resultant about the same point.

b) Prove that the centre of gravity of a uniform triangular lamina is the same as that of three equal particles placed at the vertices of the triangle.

B) Attempt any one 05

c) Three forces $\vec{P}, \vec{Q}, \vec{R}$ act along the sides BC, CA, AB of a ΔABC, taken in order. Prove that if the resultant passes through the orthocenter of the triangle ABC, then $P.\sec A + Q.\sec B + R.\sec C = 0$

d) Three rods of unequal lengths are jointed to form a ΔABC. If the masses of the sides a, b, c be proportional to (b + c - a), (c + a - b) and (a + b - c), prove that the centre of gravity is the incentre.

Q.4 Choose the correct alternative and rewrite the sentence. 10

i) If X and Y are the resolved parts of force $\vec{R}$ along OX and OY respectively where OX⊥OY then $R^2 =$ ----------

a) $X^2 - Y^2$ b) $X^2 + Y^2$ c) $\frac{X^2}{Y^2}$ d) $X^2.Y^2$

ii) If a, b, c are the lengths of sides opposite to the angles A, B, C respectively of ΔABC then $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ in known as -----------

a) Sine rule b) Cosine rule
c) Lami's theorem d) Triangle law

iii) Centre of gravity of rigid body is ------------

a) Unique b) Non unique
c) A line d) None of the above

iv) Two equal, unlike, parallel forces acting at the different points of the rigid body are said to form ------------

a) A triangle b) A parallelogram
c) A couple d) A square

v) If Δ and S denotes area of triangle and half of perimeter of the triangle respectively then $\frac{\Delta}{s} =$ ---------

a) Radius of incircle of triangle
b) Radius of circum circle of triangle
c) Perimeter of triangle
d) Area of incircle of triangle

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SUBJECT CODE NO:- B-2047
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem-V) Examination OCT/NOV 2019
Mathematics MAT-503 1) Mathematical Statistics - I

[Time: 1:30 Hours] **[Max. Marks:50]**

Please check whether you have got the right question paper.

N.B
i) All questions are compulsory.
ii) Figures to the right indicate full marks.
iii) Calculator is allowed.

Q.1 (A) Attempt any one:

(a) Prove that the sum of the squares of the deviations of all the values taken about their arithmetic mean is minimum. 08

(b) State and prove the formula for mode in case of continuous frequency distribution. 08

(B) Attempt any one:

(c) Find the median for the following distribution: 07

Class	0-10	10-20	20-30	30-40	40-50
Frequency	14	25	27	24	15

(d) Find mean and standard deviation of first 'n' natural members. 07

Q.2 (A) Attempt any one:

(a) Prove that the root mean square deviation is least when deviations are measured from the mean. 08

(b) Define moments. Establish the relationship between moments about mean and the moments about any point. 08

(B) Attempt any one

(c) Find, mean and standard deviation if the first three moments of a distribution about the value 5 of variable are 5, 20 and 40. 07

(d) Calculate first and second moments about zero for the observations 3, 8, 11, 12, 20. 07

Q.3(A) Attempt any one:

(a) If A,B,C are pairwise independent and A is independent of BUC, then prove that A, B, C are mutually independent. 05

(b) Prove that, for three observations x_1, x_2 and x_3: 05

$$AH = G^2$$

Where A = arithmetic mean,
H= harmonic mean,
G= geometric mean

(B) Attempt any one:

(c) Find the constant K for the probability density function 05
$f(x) = K.x^2, 0 \leq x \leq 3$
= 0, elsewhere
and compute $P(1 \leq x \leq 2)$.

(d) From a bag containing 5 white, 7 red, and 4 black balls, a man draws 3 at random. Find the probability of being all white. 05

Q.4 Choose correct alternative of the following: 10

(i) Which of the following is a continuous variable.
(a) The number of children's in a family.
(b) temperature
(c) The number of students in a class.
(d) The number of workers in a firm.

(ii) The mode of the distribution

x	10	20	30	40	50
f	4	5	7	6	3

is……….
(a) 40 (b) 50 (c) 60 (d) 30

(iii) The skewness of the distribution 4,4,5,5 is………….
(a) 0 (b)1 (c) -1 (d) 2

(iv) Total number of possible cases when three dice are thrown simultaneously is……….
(a) 36 (b) 81 (c) 216 (d) 253

(v) The value of the variable which divides a series into two equal parts so that one half or more of the items are equal to or less than it is …………
(a) mode (b) median (c) mean (d) geometric mean

OR

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SUBJECT CODE NO:- B-2047
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem-V) Examination OCT/NOV 2019
Mathematics 504 2) Ordinary Differential Equation –I

[Time: 1:30 Hours] **[Max. Marks:50]**

Please check whether you have got the right question paper.

N.B
i) All questions are compulsory.
ii) Figures to the right indicate full marks.

Q.1(A) Attempt any one: 08

(a) If r is a root of multiplicity m of apolynomial p, deg $p \geq 1$, then prove that
$p(r) = p^{'}(r) = \cdots = p^{(m-1)}(r) = 0$
and $p^m(r) \neq 0$.

(b) Consider the equation 08
$y^{'} + ay = b(x)$
Where a is a constant, and b is a continuous function on interval I, If x_0 is a point in I and C is any constant, then prove that the function ϕ defined by

$$\phi(x) = e^{-ax} \int_{x0}^{x} e^{at}\, b(t)dt + ce^{-ax}$$

is a solution of this equation. Also prove that every solution has this form.

(B) Attempt any one: 07

(c) If ϕ is solution of the differential equation
$y^{'} + iy = x$
Such that $\phi(0) = 2$.
Find $\phi(\Pi)$.

(d) Find all roots of the polynomial 07
$z^3 + 24$.

Q.2(A) Attempt any one:

(a) For any real x_0, and constants $\propto, \beta$, Prove that there exists a solution $\emptyset$ of the initial value problem 08

$$L(y) = y" + a_1 y' + a_2 y = 0$$
$$y(x_0) = \propto, y^{'}(x_0) = \beta$$

on $-\infty < x < \infty$

(b) Prove that two solutions ϕ_1, ϕ_2 of 08
$L(y) = y'' + a_1 y' + a_2 y = 0$
are linearly independent on an interval
I if and only if,
$W(\phi_1, \phi_2) \neq 0$
For all x in I

(B) Attempt any one: 07

(c) Find all solutions ϕ of the differential equation
$y'' + y = 0$
Satisfying
$\phi(0) = 1, \phi(\Pi/2) = 2$

(d) Find all solutions of the equation 07
$y'' + 4y = cosx$

Q.3(A) Attempt any one: 05

(a) If ϕ_1, ϕ_2 are solutions of
$L(y) = y'' + a_1 y' + a_2 y = 0$
on an interval I containing a point x_0, then prove that
$W(\phi_1, \phi_2)(x) = e^{-a_1(x-x_0)} W(\phi_1, \phi_2)(x_0)$

(b) If $\propto, \beta$ are any two constants and x_0 is any real number. On any interval I containing x_0, prove that there exists at must one solution ϕ of the initial value problem 05
$L(y) = y'' + a_1 y' + a_2 y = 0$
$y(x_0) = \propto, y'(x_0) = \beta$

(B) Attempt any one: 05

(c) Consider the equation
$y' + 5y = 2$
(i) Show that the function ϕ given by
$$\phi(x) = \frac{2}{5} + Ce^{-5x}$$
is solution, where C is constant.
(ii) Assuming every solution has this form, find the solution satisfying $\phi(1) = 2$.

(d) Show that the functions: 05
$\phi_1(x) = cosx, \phi_2(x) = sinx$
are linearly independent for $-\infty < x < \infty$.

Q.4 Choose the correct alternative 10

(1) Every polynomial of degree 3 with real coefficient has at least……….
(a) Two real roots (b) one real root
(c) Three real roots (d) None of these

(2) The function ϕ is called solution of $y^{'} = f(x, y)$, where $x, y \epsilon s, if$ …..
(a) $\phi(x)$ is in S (b)$\phi^{'}(x) = f(x, \phi(x))$
(c) both (a) and (b) (d) None of these

(3) The functionsϕ_1, ϕ_2 defined by $\phi_1(x) = x \; and \; \phi_2(x) = |x|$ are……..
(a) Linearly independent
(b) Linearly dependent
(c) Both (a) and (b)
(d) None of these

(4) Let b be continuous on an interval I. Every solution$\Psi \; of \; L(y) = y" + a_1 y' + a_2 y = b(x)$on I can be written as $\Psi = C_1\phi_1 + C_2\phi_2 + \Psi_P$then Ψ_P is known as …………
(a) Complementary function
(b) Basic solution
(c) Particular solution
(d) None of these

(5) The solution $\phi \; of \; y^{'} + ay = 0$ is given by……….
(a) $\phi(x) = ce^{-ax}$
(b) $\phi(x) = ce^{ax}$
(c) $\phi(x) = e^{ax}$
(d) None of these

OR

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SUBJECT CODE NO:- B-2047
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem-V) Examination OCT/NOV 2019
Mathematics 505 3) Programming in C - I

[Time: 1:30 Minutes] **[Max. Marks:40]**

Please check whether you have got the right question paper.

N.B
i) All questions are compulsory.
ii) Assume the data wherever not given with justification.
iii) Figures to the right indicate full marks.

Q.1 (A) Attempt any one: 05

(a) Write the rules to define a symbolic constant in # define statement.
(b) Explain structure of C programs.

(B) Attempt any one: 05

(c) Write a C program for storage classes.
(d) Write a C program to calculate the average of a set of N numbers.

Q.2 (A) Attempt any one: 05
(a) Discuss relational operators in C language with example.
(b) Explain arithmetic operators and integer operators.

(B) Attempt any one: 05
(c) Write a program to compute Salesman's salary with suitable data.
(d) Write a program to solve the quadratic equation $ax^2 + bx + c = 0$.

Q.3 (A) Attempt any one: 05

(a) Explain the uses of following functions
(i) islower
(ii) toupper
(iii) tolower
(b) Explain mixed data output using printf function

(B) Attempt any one: 05
(c) Write a program for the function % ( ) specification.
(d) Write a program to detect errors in scanf input.

Q.4 Fill in the blanks and write the complete sentence. 10

(a) A # define is a……….. Compiler and not a ……………
(b) Identifiers must not contain ………………
(c) The logical operator && means …………
(d) The ………..function requires a set of parentheses.
(e) % [   ] is used to read strings with ……….spaces.