Total No. of Printed Pages:2

SUBJECT CODE NO:- L-2011
FACULTY OF SCIENCE
B.Sc. T.Y. (Sem-V) Examination Oct/Nov 2018
Mathematics MAT - 501
Real Analysis – I

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
i) All questions are compulsory.
ii) Figures to the right indicate full marks.

Q.1 A. Attempt any one:

a) If $f: A \rightarrow B$ and $X \subset A$, $Y \subset A$, then prove that 08

$$f(X \cup Y) = f(x) \cup f(y)$$

b) If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent to L, then prove that $\{s_n\}_{n=1}^{\infty}$ Cannot also coverage to a limit distinct from L. 08

B. Attempt any one:

c) Define the composition of functions. 07
if $f(x) = 1 + \sin x \quad (-\infty < x < \infty)$
$g(x) = x^2 (0 \leq x < \infty)$, then find gof

d) Prove that 07

$$\lim_{n \to \infty} \frac{2n}{n + 4\, n^{1/2}} = 2,$$

By $\in -\delta$ *method*

Q.2 A. Attempt any one: 08

a) Prove that a non-decreasing sequence which is bounded above is convergent.

b) If $\{s_n\}_{n=1}^{\infty}$ is a couchy sequence of real numbers, then prove that $\{s_n\}_{n=1}^{\infty}$ is bounded.

B. Attempt any one: 07

c) $if\ u_1 = \frac{x_2 x_3}{x_1}, \quad u_2 = \frac{x_1 x_3}{x_2}, \quad u_3 = \frac{x_1 x_2}{x_3}$,
Then prove that $J(u_1, u_2, u_3) = 4$

d) $if\ u = x^2 + y^2 + z^2, \quad v = x + y + z,\ w = xy + yz + zx$, Show that the Jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ vanishes identically.

Q.3 A. Attempt any one:

a) If $0 < x < 1$, then prove that $\sum_{n=0}^{\infty} x^n$ coverage's $\frac{1}{(1-x)}$. 05

b) $if\ \sum_{n=1}^{\infty} b_n$ coverage's absolutely and $\lim_{n\to\infty} {|a_n|}/{|b_n|}$ exists, then prove that $\sum_{n=1}^{\infty} a_n$ converges absolutely. 05

B. Attempt any one:

c) Show that $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ Diverges. 05

d) Show that $\sum_{n=1}^{\infty} {1}/{n^2}$ is convergent. 05

Q.4 Choose the correct alternative and rewrite the sentence. 10

i) The graph of the function f is the subset of $X \times Y$ is the set……

a) $\{(x, f(x))/\ x \in X\}$
b) $\{(x, x)\ /\ x \in X\}$
c) $\{(f(x), x)/\ x \in X\}$
d) $\{(x, x^2)/\ x \in X\}$

ii) All subsequences of a convergent sequence of real numbers converges to--------

a) Different limits b) infinity c) same limit d) Zero

iii) If $\{S_n\}_{n=1}^{\infty}\ where\ s_n = (-1)^n$ then

$\lim_{n\to\infty} Sup\ S_n$ =----------------

0 b) -1 c) n d)1

iv) If $\sum_{i=1}^{\infty} a_i$ coverage's to s then $\sum_{i=2}^{\infty} a_i$ converges to-----------

a) a_1 b) a_2 c) $s - a_1$ d) $s + a_1$

v) $\sum_{n=1}^{\infty} a_n$ converges absolutely if-----------------

a) $\sum_{n=1}^{\infty} a_n$ Converges.

b) $\sum_{n=1}^{\infty} |a_n|$ Converges.

c) $\sum_{n=1}^{\infty} a_n$ Diverges.

d) $\sum_{n=1}^{\infty} |a_n|$ Diverges.

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SUBJECT CODE NO:- L-2012
FACULTY OF SCIENCE
B.Sc T.Y. (Sem-V) Examination Oct/Nov 2018
Mathematics MAT - 502
Abstract Algebra – I

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
i) All questions are compulsory.
ii) Figures to the right indicate full marks.

Q.1 (A) Attempt any one: 08

a) $if\ G$ Is a group, H a subgroup of G; for a,$b \in G$ is congruent to b modulo H written as a$\equiv b$ mod H if $ab^{-1} \in H,$ then prove that a $\equiv b\ mod\ H$ is an equivalence relation.
b) $if\ \emptyset$ is a homomorphism of G onto $\bar{G}$ with kernel K, then prove that G/K $\approx \bar{G}$

(B) Attempt any one: 07

c) If H is subgroup of G, then show that there is a one to one correspondence between the set of left cosets of H in G and the set of right cosets of H in G.
d) If G is the group of all 2×2 matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $ad - bc \neq 0$ under the matrix multiplication. If $\bar{G}$ is the group of all non-zero real numbers under multiplication. Define $\emptyset: G \to \bar{G}$ by $\emptyset \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$. Prove that $\emptyset$ is a homomorphism of G onto $\bar{G}$. Also determine the kernel of this homomorphism.

Q.2 (A) Attempt any one: 08

a) If R is a communicative ring with unit element and M is an ideal of R, and if M is a maximal ideal of R the prove that R/M is a field.
b) For given two polynomials $f(x) and g(x) \neq 0\ in\ F[x]$, prove that there exists two polynomials t(x) and r(x) in F[x] such that $f(x) = t(x)g(x) + r(x)$ where $r(x)$=0 or $\deg r(x) < \deg g(x)$.

B) Attempt any one: 07

c) If $U\ and\ V$ are ideals of a ring R, and if
$U + V = \{u + v | u \in U, v \in V\},$
$then\ show\ that\ U + V\ is\ also\ an\ ideal\ of\ R$
d) If R is a ring with unit element. R not necessarily commutative such that only right-ideals are (0) and R itself, then prove that R is a Division Ring.

Q.3 (A) Attempt any one: 05

a) If G a group then prove that every $a \in G$ has unique inverse in G.
b) If p is prime number then prove that J_p , then ring of integers $mod\ p$ is a field.

(B) Attempt any one: 05

c) If G is a group of even order, then prove that it has an element $a \neq e$ satisfying $a^2 = e$

d) Prove that any field is an integral domain.

Q.4 Choose the correct alternative and rewrite the sentence: 10

1) The symmetric group S_3 of degree 3 has……………
   - a) 6 elements
   - b) 3 elements
   - c) 1 element
   - d) 9 elements

2) The set $G = \{1, w, w^2\}$, where w is a cube root of unity , is a finite group with respect to usual multiplication then inverse of w^2 is …………….
   - a) 1
   - b) w
   - c) w^2
   - d) -1

3) If N is a normal subgroup of a group G then for all $x, y \in G$, we have $Nx\ Ny =$ ………
   - a) Nx
   - b) Ny
   - c) Nxy
   - d) xNy

4) The concept of left and right ideals coincides in ………..
   - a) group
   - b) alelian group
   - c) non-commutative ring
   - d) Communicative ring.

5) The ring $(\{0,1,2, \dots, p-1\}, +_p, \times_P)$is a fields if p is………
   - a) 4
   - b) 5
   - c) 6
   - d) 8

Total No. of Printed Pages:02

SUBJECT CODE NO:- L-2027
FACULTY OF SCIENCE
B.Sc. F.Y. (Sem-I) Examination Oct/Nov 2018
Mathematics MAT - 101
Differential Calculus

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
1) Attempt all questions.
2) Figures to the right indicate full marks.

Q.1 A) Attempt any one: 08

a) Show that $f'(c)$ is the tangent of the angle which the tangent line to the curve $y = f(x)$ At the point P[c,f(c)] makes with X-axis.

b) If $y = e^{ax}\sin(bx + c)$, then show that
$\frac{d^n y}{dx^n} = (a^2 + b^2)^{\frac{n}{2}}\ e^{ax}\sin(bx + c + n\Phi)$ where $\Phi = \tan^{-1}(\frac{b}{a})$.

B) Attempt any one: 07

c) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$, prove that $x^2 y_{n+2} + (2n + 1)xy_{n+1} + 2n^2 y_n = 0$.

d) Find the n^{th} derivative of : $y = \frac{x^2}{(x+2)(2x+3)}$

Q.2 A) Attempt any one: 08

a) If a function f is (i) continuous in a closed interval [a,b] and (ii) derivable in the open interval] a,b[, then prove that there exists at least one value $c\ \epsilon\ ]a, b[$ such that
$\frac{f(b)-f(a)}{b-a} = f'(c)$

b) If $Z = f(x, y)$ is a homogeneous function of x,y of degree n then prove that
$x\ \frac{\partial z}{\partial x} + y\ \frac{\partial z}{\partial y} = nz$
For all $x, y\ \epsilon$ the domain of the function.

B) Attempt any one: 07

c) If in the Cauchy's mean value theorem, $f(x) = e^x$ and $F(x) = e^{-x}$, show that c is arithmetic mean between a and b.

d) If $u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}; xy \neq 0,$ prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2-y^2}{x^2+y^2}$.

Q.3 A) Attempt any one: 05

a) Prove that
$\text{div}(\Phi\vec{f}\,) = \Phi\, div\, \vec{f} + \vec{f}\,.grad\, \Phi$

b) Prove that div $\vec{f}$ and curl $\vec{f}$ are point functions.

B) Attempt any one: 05

c) If $\vec{f} = x^2 z\,\vec{\imath} - 2y^3z^2\,\vec{\jmath} + xy^2\overrightarrow{zk},$ find curl $\vec{f}$ at (1,-1,1).

d) For every $x\epsilon R$, prove that $\cosh^2 x + \sinh^2 x = cosh2x$

Q.4 Choose the correct alternative: 10

i) If $y = sechx$, then $\frac{dy}{dx} = -----$
a) $tanhx\ \ sechx$ b) $-tanhx\ sechx$ c) $-cosech^2 x$ d) $cosech^2 x$

ii) If $y = \cos(3x+5)$, then $y_3 = ----$
a) $3^2\cos(3x+5+\frac{3\pi}{2})$ b) $3^3\cos(3x+5+3\pi)$
c) $3^3\cos(3x+5+\frac{3\pi}{2})$ d) None of these

iii) For all $x\epsilon R$, cos x = --------------
a) $1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+----$ b) $1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\ ----$
c) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ ----$ d) $x+\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ ----$

iv) div $\vec{r}$ = ------------
a) 0 b) 3 c) 2 d) 1

v) The gradient of a scalar point function is -----------
a) Scalar point function b) Scalar unit function
c) Vector point function d) None of these

Total No. of Printed Pages:02

SUBJECT CODE NO:- L-2028
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. F.Y. (Sem-I) Examination Oct/Nov 2018
Mathematics MAT - 102
(Differential Equations)

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
1) Attempt all questions.
2) Figures to the right indicates full marks.

Q.1 A) Attempt any one: 08

a) Explain the method of solving the differential equation $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x or constants.

b) With usual notations, prove that

$\frac{1}{f(D)} x.V = \left\{x - \frac{1}{f(D)}.f'(D)\right\}\frac{1}{f(D)}.V$

Where V be any function of x.

B) Attempt any one: 07

c) Solve the simultaneous equations:

$\frac{dx}{dt} - 7x + y = 0,$

$\frac{dy}{dt} - 2x - 5y = 0.$

d) Solve $(D^2 + 1)y = xe^{2x}$

Q.2 A) Attempt any one: 08

a) Explain the method of solving the differential equation.

$$x^n \frac{d^n y}{dx^n} + P_1 x^{n-1} \frac{d^{n-1}y}{dx^{n-1}} + ------ + P_{n-1} + x.\frac{dy}{dx} + P_n y = X$$

Where $P_1, P_2, \dots \dots \dots . P_n$ are constants and X is a function of x.

b) Explain the method of solving the differential equation.

$\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1}y}{dx^{n-1}} + ---- + P_n.y = X$

Where $P_1, P_2, ------ P_n$ are constants and X is a function of x.

B) Attempt any one: 07

c) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 3.e^{5/2\ x}$

d) Solve $x^2\frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 4y = x^4$.

Q.3 A) Attempt any one: 05

a) Derive the partial differential equation by eliminating an arbitrary function Φ from $\Phi(u, v) = 0$, where u and v are functions of x,y,z.

b) Explain the method of solving the equation of the form $\frac{d^2y}{dx^2} = f(y)$

B) Attempt any one: 05

c) Solve: $\cos^2 x\frac{dy}{dx} + y = \tan x$

d) Form a partial differential equation by eliminating a and b from $z = a(x + y) + b$.

Q.4 Choose the correct alternative: 10

i) The condition for the differential equation $Mdx + Ndy = 0$ b . to be exact is------

a) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ b) $\frac{\partial M}{\partial y} = \frac{-\partial N}{\partial x}$ c) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ d) $\frac{\partial M}{\partial x} = \frac{-\partial N}{\partial y}$

ii) The integrating factor of the differential equation $(x^2 + 1)\frac{dy}{dx} + 2xy = 4x^2$ is -----

a) $\log(x^2 + 1)$ b) $x^2 + 1$ c) $\frac{1}{x^2+1}$ d) None of the these

iii) A partial differential equation must contain ----------

a) At least two independent variables b) only one independent variable

c)one independent and one dependent variable d) None of the above

iv) The particular integral of the differential equation $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$ is ----

a) $\frac{x^4}{5}$ b) $\frac{x^4 \log x}{5}$ c) log x d) $\frac{x^4}{\log x}$

v) The partial differential equation obtained by eliminating arbitrary constants a and b from the equation z= (x+a)(y+b) is -------

a) z = pq b) z = p+q c) $z = \frac{p}{q}$ d) p-q = z

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SUBJECT CODE NO:- L-2033
FACULTY OF SCIENCE
B.Sc. S.Y. (Sem-III) Examination Oct/Nov 2018
Mathematics MAT – 301
Number Theory

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
1. Attempt all questions.
2. Figures to the right indicate full marks.

Q.1 A) Attempt any one

a) If a and b are integers not both zero then prove that there exist integers x and y such that gcd (a, b) =ax + by — 08

b) Prove that linear Diophantine equation ax + by =c has a solution if and only if d/c ,where d= gcd (a, b) If x_o , y_0 is any particular solutions of this equation, then show that all other solutions are given by

$$x = x_0 + \left(\frac{b}{d}\right)t \qquad y = y_0 - \left(\frac{a}{d}\right)t$$

where t is an arbitrary integers

B) Attempt any one — 07

c) By using Euclidean algorithm, obtain integers x and y satisfying $\gcd(56,72) = 56x + 72y$

d) Find L.C.M (143,227)

Q.2 A) Attempt any one — 08

a) If $n > 1$ be a fixed integer and a, b, c, d be arbitrary integers then prove that

i) $if\ a \equiv b(modn)\ and\ b \equiv c\ (modn)\ then\ show\ that\ a \equiv c(modn)$

ii) $if\ a \equiv b(modn), then\ a^k \equiv b^k(modn)$

b) If P be a prime and suppose that $P \times a$. Then prove that $a^{p-1} \equiv 1(mod\ P)$

B) Attempt any one — 07

c) Show that Wilson's theorem. $(P-1)\,! \equiv -1(mod\ P)$ is true for P=13

d) Show that 41 divides $2^{20} - 1$

Q.3 A) Attempt any one — 05

a) If $n = P_1^{k_1}\ P_2^{k_2} - - - - P_r^{k_r}$ is the prime fraction of n >1 , then prove that

$$\sigma\,(n) = \frac{{P_1}^{k_1+1} - 1}{p_1 - 1} . \frac{{P_2}^{k_2+1} - 1}{P_2 - 1} - - - - - \frac{{P_r}^{k_r+1} - 1}{P_r - 1}$$

b) If P is a prime and k>0 , then show that $\emptyset(P^k) = P^k(1 - \frac{1}{P})$

B) attempt any one 05

c) calculate $\emptyset(1001)$

d) if n is odd integer , then show $\emptyset(2n) = \emptyset(n)$

Q.4 Choose the correct alternatives 10

i) $l$cm(12, 30) is ______

a) 30 b) 12 c) 60 d) 120

ii) For any two integers a & b $a \equiv b(mod\ n)$ means________

a) n divides a + b
b) n divides a – b
c) n divides ab
d) none of these

iii) number of possible solution for linear congruence $18x \equiv 30(mod\ 42)$ are ______

a) 6 b) 1 c) 4 d)3

iv) If p & q are distinct primes with $a^p \equiv a(mod\ q)$ and $a^q \equiv a(mod\ p)$ then $a^{pq} \equiv a(mod ------)is$

a) P b) q c) pq d) $^p/_q$

v) Value of $\tau(12)$ is ______.

a) 6 b) 5 c) 4 d) 3

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SUBJECT CODE NO:- L-2034
FACULTY OF SCIENCE
B.Sc. S.Y. (Sem-III) Examination Oct/Nov 2018
Mathematics MAT - 302
Integral Transforms

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
I. All questions are compulsory.
II. Figures to right indicate full marks.

Q.1 A) Attempt any one: 08

a) Prove that $\beta(l, m) = \frac{\Gamma(l)\,\Gamma(m)}{\Gamma(l+m)}$, where m, l are positive integer.

b) Define Laplace transform and Fourier transform , find L{1} and F(1)

B) Attempt any one: 07

a) Prove that

$$\int_0^1 \frac{x^2 dx}{(1-x^4)^{\frac{1}{2}}} \times \int_0^1 \frac{dx}{(1+x^4)^{\frac{1}{2}}} = \frac{\pi}{4\sqrt{2}}$$

b) Prove that $\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{2}{n}\right)\ \Gamma\left(\frac{3}{n}\right), ------, \Gamma\left(\frac{n-1}{n}\right) = \frac{2\pi^{\left(\frac{n-1}{2}\right)}}{n^{\frac{1}{2}}}$, where n is an integer.

Q.2 A) Attempt any one : 08

a) If L{F(t)}=f(s) and

G(t)= $\begin{cases} F(t-a), if\ t > a; \\ 0, \quad if\ t < a, \end{cases}$

Then Prove that L {G (t)}=e^{-as}f(s).

b) If L {F(t)}=f(s), then prove that

L$\{t^n F(t)\} = (-1^n)\frac{d^n f(s)}{ds^n}$.

B) Attempt any one : 07

c) Solve by using Laplace transform

$$(D^2 + 9)y = 6\cos(3t), with\ y(0) = 2, y'(0) = 0$$

d) Find L {$e^{-2t}[3\sin(2t) - 31\cos h(2t)]$}

Q.3 A) Attempt any one: 05

a) If $L^{-1}\{f(s)\} = F(t), then\ prove\ that$
$L^{-1}\{f(as)\} = \frac{1}{a} F\left(\frac{t}{a}\right), a > 0.$

b) Prove that

$$\int_0^\infty \left(\frac{sint}{t}\right) dt = \frac{\pi}{2}$$

B) Attempt any one: 05

c) Find the Fourier transform of

$$f(x) = \begin{cases} x, if\ |x| \leq | \\ 0, if\ |x| > | \end{cases}$$

d) Find the Fourier transform of $f(x) = e^x$

Q.4 For each of the following questions four alternatives are given for the answers. Only one of them is correct. 10

Choose the correct alternatives.

1) The value of $\Gamma(4)$ is ______
a) 1 b) 24 c)∞ d) none of these

2) $L\{e^t\ t^n\} = - - - - -$
a) $\frac{n!}{(s-1)^{n+1}}$ b) $\frac{1}{s^n}$ c) $\frac{1}{s^{n-1}}$ d) S^{n+1}

3) The value of $\Gamma\left(\frac{9}{10}\right)$ is ______
a) 32 b) 64 c) 16 d) none of these

4) The Fourier transform of $f(x) = \frac{1}{x}$ is ______
a) $\frac{s}{s^2+a^2}$ b) $\frac{a}{s^2+a^2}$ c) $\frac{s}{s^2-a^2}$ d) none of these

5) The Laplace transform of $\frac{1}{a^2}$ (1- sinh2t) is_______
a) $\frac{1}{s}$ b) $\frac{s}{(s^2+4)^2}$ c) $\frac{s+1}{s-1}$ d) none of these

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SUBJECT CODE NO:- L-2037
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y. (Sem-V) Examination Oct/Nov 2018
Mathematics MAT-503
1) Mathematical Statistics – I

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
1) All questions are compulsory.
2) Figures to the right indicate full marks.
3) Calculator is allowed.

Q.1 A) Attempt any one:

a) Explain "ogive" with suitable example. 08

b) State and prove the formula for mean of combined distribution. 08

B) Attempt any one:

c) Find the harmonic mean of the marks obtained by 25 students in a class test are given below: 07

Marks obtained	11	12	13	14	15
No. of students	3	7	8	5	2

d) Calculate the weighted mean of first n natural numbers whose weights are equal to the squares of the corresponding number. 07

Q.2 A) Attempt any one:

a) State and prove the additive law of probability. 08

b) Discuss the merits and demerits of mean, median and mode. 08

B) Attempt any one:

c) Find the mean deviation from mean and standard deviation of the series $a, a+d, a+2d, ---, a+nd$. 07

d) The mean marks of 100 students were found to be 40, later on it was discovered that a score of 53 was misread as 83. Find the corrected mean corresponding to the corrected score. 07

Q.3 A) Attempt any one:

a) Define the following terms: 05
   i) Variance
   ii) Median
   iii) Mode
   iv) Moments
   v) Probability

b) Show that the deviations of values $x_i, (i = 1, 2, \ldots \ldots n)$ from their mean $\bar{x}$ is zero, f_i being the frequency of x_i. 05

B) Attempt any one:

c) Write down the sample space when two dice are thrown, hence find the probability of obtaining a total of more than 10. 05

d) Find the mean deviation of 7, 9, 14, 24, 26 measured from their arithmetic mean. 05

Q.4 Choose correct alternative of the following: 10

1) The coefficient of variation when variance = 4 and mean = 40 is
   a) 4 b) 60 c) 7 d) 5

2) If A and B are independent events, with $P(A) = 0.5$ and $P(B) = 0.3$, then $P(A \cup B)$ is equal to
   a) 0.80 b) 0.70 c) 0.65 d) 0.50

3) The median of 10, 11, 12, 32, 45, 50, 2, 11, 10 is
   a) 10 b) 11 c) 12 d) 2

4) If a random variable takes at most a countable number of values, then it is called -----------
   a) Continuous random variable b) Discrete random variable
   c) Bivariate random variable d) Multivariate random variable

5) If $f(x) = K\,x\,e^{-x}, (0 \le x \le \infty)$ be a continuous distribution then, value of constant k is
   a) 4 b) 3 c) 2 d) 1

OR

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SUBJECT CODE NO:- L-2037
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y. (Sem-V) Examination Oct/Nov 2018
Mathematics MAT- 504
2) Ordinary Differential Equation –I

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Attempt <u>any one</u>: 08

a) Consider the equation
$y' + ay = b(x)$
Where a is constant and b is a continuous function on an interval I. if x_0 is a point in I and C is any constant, then prove that the function
$\phi(x) = e^{-ax} \int_{x_0}^{x} e^{at}\, b(t)dt + c\, e^{-ax}$
Is a solution of this equation?

b) Suppose that a and b are continuous function on an interval I. let A be a function such that $A' = a$. Prove that the function.
$$\psi(x) = e^{-A(x)} \int_{x_0}^{x} e^{A(t)}\, b(t)dt, \quad x_0 \epsilon I$$
Is a solution of the equation
$y' + a(x)y = b(x)$ on I.

B) Attempt any one: 07

c) Consider the equation
$L\, y' + R\, y = E\, e^{iwx}$
Where L, R, E, ω are positive constants. Find the solution ϕ which satisfies $\phi(o) = 0$

d) Suppose ϕ is a function with a continuous derivative on $0 \leq x \leq 1$ satisfying there
$\phi'(x) - 2\,\phi(x) \leq 1$and $\phi(o) = 1$. Show that $\phi(x) \leq \frac{3}{2} e^{2x} - \frac{1}{2}$

Q.2 A) Attempt any one: 08

a) Obtain the characteristic polynomial for the equation
$L\,[y] = y'' + a_1 y' + a_2 y = 0$
Where a_1, a_2 are constants. If ϕ_1, ϕ_2 are solutions of $L[y] = 0$ then prove that the function
$\phi = c_1\phi_1 + c_2\phi_2$
Is a solution of $L[y] = 0$

b) Prove that the solutions ϕ_1, ϕ_2 of $L[y] = y'' + a_1 y' + a_2 y = 0$ are linearly independent on an interval I if and only if $W(\phi_1, \phi_2)(x) \neq 0, \forall\, x \in I$.

B) Attempt any one: 07

c) Find the solutions of the initial value problem
$y'' - 2y' - 3y = 0, \qquad y(o) = 0, y'(o) = 1$

d) Compute the solution ϕ of the equation $y'' + y' - 6y = 0$ satisfying $\phi(o) = 1, \phi'(o) = 0.$

Q.3 A) Attempt any one: 05

a) If ϕ_1, ϕ_2 are two solutions of
$L[y] = y'' + a_1 y' + a_2 y = 0$
On an interval I containing a point x_0, then prove that
$W(\phi_1, \phi_2)(x) = e^{-a_1(x-x_0)}\, W(\phi_1, \phi_2)\,(x_0)$

b) Let b be continuous function on an interval I. prove that every solution ψ of
$L\,[y] = y'' + a_1 y' + a_2\, y = b(x)$ on I can be written as
$\psi = \psi_p + c_1\phi_1 + c_2\phi_2$
Where ψ_p is particular solution, ϕ_1, ϕ_2 are two linearly independent solutions of $L[y] = 0$

B) Attempt any one: 05

c) Find all solutions of the equation
$y'' - y' - 2y = e^{-x}$

d) Find all solutions of the equation
$y'' + 4y = \cos x$

Q.4 Choose correct alternative and rewrite the sentence : 10

1) The solution of the equation $y' + 3y = 0$ is
   a) $\phi(x) = c\, e^{-3x}$
   b) $\phi(x) = c\, e^{3x}$
   c) $\phi(x) = 3\, e^{-ax}$
   d) $\phi(x) = 3\, e^{ax}$

2) The equation $y' = -a(x)y$ is
   a) Nonhomogeneous
   b) Homogeneous
   c) Both (a) & (b)
   d) None of these

3) The solution of nonhomogeneous equation consists of
   a) Complementary function
   b) Particular solution
   c) Both (a) & (b)
   d) None of these

4) The characteristic polynomial of $y'' + y' - 2y = 0$ is
   a) $r^2 - r + 2$
   b) $r^2 + r + 2$
   c) $r^2 + r - 2$
   d) $r^2 - r - 2$

5) Which of the following is an initial value problems
   a) $y'' - y = 0, \quad y(o) = 0, \quad y'(1) = 2$
   b) $y'' - y = 0, \quad y(o) = 0, \quad y(1) = 1$
   c) $y'' - y = 0,$
   d) $y'' - y = 0, \quad y(o) = 0, \quad y'(o) = 1$

OR

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SUBJECT CODE NO:- L-2037
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y. (Sem-V) Examination Oct/Nov 2018
Mathematics MAT- 505
3) Programming in C – I

[Time: 1:30 Hours] **[Max.Marks:40]**

Please check whether you have got the right question paper.

N.B
1) All questions are compulsory.
2) Assume the data wherever not given with justification.
3) Figures to the right indicate full marks.

Q.1 A) Attempt any one: 05

a) Discuss data types in C language.
b) Discuss character set in C language.

B) Attempt any one: 05

c) Write a program in C to add two numbers.
d) Write a program to represent integer constant on a 16-bit computer.

Q.2 A) Attempt any one: 05

a) Explain conditional operators and bitwise operators.
b) Explain logical operators with example.

B) Attempt any one: 05

c) Write the rules for evaluation of expression in C language.
d) Write a program to find the roots of the equation
$ax^2 + bx + c = 0$

Q.3 A) Attempt any one: 05

a) Explain output of integer numbers using printf function with examples.
b) Explain printing of strings with example.

B) Attempt any one: 05

c) Write a program to read integers.
d) Write a program for getting formatted output of integers.

Q.4 Fill in the blanks and write the complete sentence. 10

a) The words int and float are called the ---------
b) In variables --------- space is not allowed.
c) The complement of relational operator ! = is --------
d) The ---------- contains the format of data being received.
e) The ------- function is used to flush out the unwanted characters.

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SUBJECT CODE NO:- L-2074
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. S.Y (Sem-III) Examination Oct/Nov 2018
Mathematics MAT - 303
Mechanics-I

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
1) Attempt all questions.
2) Figures to the right indicate full marks.
3) Draw well-labeled diagrams wherever necessary.

Q.1 A) Attempt any one 08

a) Find the magnitude and direction of the resultant of any number of coplanar forces acting at a point.

b) Prove that if any number of forces, acting on a particle, be represented in magnitude and direction, by the sides of a polygon, taken in order, then the forces are in equilibrium.

B) Attempt any one 07

c) Three forces of magnitudes equal to 1 kg, 6 kg and 9 kg act in the directions of AB, AC and AD respectively of a square ABCD. Find the magnitude of the resultant force.

d) A partical is placed at the centre O of the circle inscribed in a ΔABC. Forces $\vec{P}, \vec{Q}, \vec{R}$ acting along $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OC}$ respectively are in equilibrium.
Prove that-
$P:Q:R = \cos\frac{A}{2}:\cos\frac{B}{2}:cos\frac{C}{2}$

Q.2 A) Attempt any one 08

a) Prove that if three forces of magnitudes P, Q and R respectively acting on a particle are in equilibrium, then each is proportional to the sine of the angle between the other two.
i.e. $\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$
where $\measuredangle(\vec{Q}, \vec{R}) = \alpha, \measuredangle(\vec{R}, \vec{P}) = \beta \ and \ \measuredangle(\vec{P}, \vec{Q}) = \gamma$

b) Prove that the necessary and sufficient condition that a given system of forces acting upon a rigid body is in equilibrium is that the force sum and moment sum must separately vanish.

B) Attempt any one 07

c) Find the angle between two equal forces P, when their resultant is equal to $\frac{P}{2}$.

d) A force $\vec{F}$ of magnitude 8 units acts at a point (2, 3, 4) along the line,

$$\frac{(x-2)}{3} = \frac{(y-3)}{4} = \frac{(z-4)}{5}$$

Find the moment of the force $\vec{F}$ about y-axis.

Q.3 A) Attempt any one 05

a) Prove that the vector moment of the resultant couple of two couples acting upon a rigid body is the sum of the vector moment of the given couples.

b) Prove that if a system of parallel forces of magnitudes $F_1, F_2, F_3, \dots \dots \dots \dots, F_n$ act at some given n points, then the resultant of these forces act through their centre.

B) Attempt any one 05

c) Find the vector moment of a force $\vec{F} = \vec{\imath} + 2\vec{\jmath} + 3\vec{k}$ acting at a point (-4, 2, 3) about the origin.

d) Three rods of unequal lengths are jointed to form a ΔABC. If the masses of the sides a, b, c be proportional to $(b + c - a), (c + a - b)\ and (a + b - c)$. Prove that the centre of gravity is the in centre.

Q.4 Choose the correct alternative and rewrite the sentence. 10

i) If $\vec{P}$ and $\vec{Q}$ are the resolved parts of a force $\vec{R}$ then angle between $\vec{P}$ and $\vec{Q}$ is ----------.
   a) 0^0 b) 90^0 c) 180^0 d) 360^0

ii) Centroid of triangle is the point of intersection of --------- of the triangle.
   a) Altitudes b) Medians
   c) Perpendicular bisectors of sides d) Angle bisectors

iii) Forces forming a couple produces only a motion of ----------.
   a) Translation b) Rotation
   c) Translation and rotation d) Neither translation nor rotation

iv) Centre of gravity of the uniform rod is at its -------------.
   a) End points b) Mid-point
   c) End points and mid-point d) All the above

v) If the force $\vec{P}$ makes an angle θ with horizontal line OX then resolved part of force $\vec{P}$ along OX is --------------

a) $P \sin \theta$
b) $P \cos \theta$
c) $P + \sin \theta$
d) $P - \cos \theta$

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SUBJECT CODE NO:- L-2147
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y. (Sem-VI) Examination Oct/Nov 2018
Mathematics MAT-601
Real Analysis-II

[Time: 1:30 Hours] **[Max.Marks:50**

Please check whether you have got the right question paper.

N.B
i) All questions are compulsory
ii) Figures to the right indicate full marks

Q.1 A) Attempt any one:

a) If G_1 and G_2 are open subsets of the metric space m ,then prove that $G_1 \cap G_2$ is also open subset of M. 08

b) Let E be a subset of the metric space M. Then prove that the point $x \in M$ is a limit point of E if and only if every open ball $B[x; r]$ about x contains at least one point of E 08

B) Attempt any one:

c) For $P \langle x_1, y_1 \rangle$ and Q $\langle x_2, y_2 \rangle$,Define $\sigma(P, Q) = |x_1 - x_2| + |y_1 - y_2|$ show that σ is a metric for the set of ordered pairs of real numbers. 07

d) Let F be the function from R^2 on to R^1 define by $f(\langle x, y \rangle) = x$ $(\langle x, y \rangle \in R^2)$ show that F is continuous on R^2 07

Q.2 A) Attempt any one:

a) Let f be a bounded function on the closed bounded interval $[a, b]$. Then prove that $f \in R[a, b]$ if and only if , for each $\in > 0$, there exist a subdivision σ of $[a, b]$ such that : $U[f; \sigma] < L[f; \sigma] + \in$ 08

b) If $f \in R[a, b]$, if $F(x) = \int_a^x f(t)dt$ $(a \leq x \leq b)$ and if f is continuous at $x_0 \in [a, b]$ then prove that $F'(x_0) = f(x_0)$ 08

B) Attempt any one

a) For $f(x) = sinx$, $l\ 0 \leq x \leq \pi/_2)$ and $\sigma_n = \{0, \frac{\pi}{2n}, \frac{2\pi}{2n}, --- \frac{n\pi}{2n}\}$. Compute $U[f; \sigma_n]$ and prove that $\lim_{n \to \infty} U[f; \sigma_n] = 1$ 07

b) Find the Fourier series of $f(x) = e^x$ in $[-\pi, \pi]$ 07

Q.3 A) Attempt any one:

a) If A is a closed subset of the compact metric space <M,ϱ> then prove that the metric space <A, ϱ > is also compact. 05

b) If the subset A of the metric space <M, ϱ > is totally bounded then prove that A is bounded 05

B) Attempt any one:

c) If $T(x) = x^2$ $(0 \leq x \leq \frac{1}{3})$, prove that T is a contraction on $[0, \frac{1}{3}]$ 05

d) Given $\in > 0$, find $\delta > 0$ such that $|sinx - sina| < \in (|x - a| < \delta; \ -\infty < a < \infty)$ 05

Q.4 Choose the correct alternative : 10

1) If ϱ is metric , for set m then________
a) 2ϱ is metric b) 2ϱ is note metric c) both a and b not true d) none of these

2) If a homeomorphism from m_1 onto m_2 exists , we say that m_1 and m_2 are _______
a) homeomorphism b) homeomorphic c) isomorphism d) All the above

3) Every finite subset of any metric space is_________
a) compact b) not compact c) not complete d) both a and c

4) $\int_{-\pi}^{\pi} sinnx\, dx =$ __________ for all n
a) π b) $-\pi$ c) 0 d) 1

5) If F is bounded function on the closed bounded interval $[a, b]$, we say that F is Riemann intergrable on $[a, b]$ if __________

a) $\int_{-a}^{b} f = \int_{a}^{b} f$ b) $\int_{-a}^{b} f \neq \int_{a}^{-b} f$
c) $\int_{-a}^{b} f \neq \int_{a}^{-b} f = 0$ d) $\int_{-a}^{b} f = -\int_{a}^{b} f$

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SUBJECT CODE NO:- L-2148
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y. (Sem-VI) Examination Oct/Nov 2018
Mathematics MAT - 602
Abstract Algebra – II

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
i) All questions are compulsory
ii) Figures to the right indicate full marks.

Q.1 A) Attempt any one 08

a) If S and T are subsets of a vector space V , then prove that:
   i) S ⊆ T implies L(S) ⊆ L(T),
   ii) L (S∪T) = L(S) + L(T)

b) If U is a vector space and W is a subspaces of U, then prove that there is a homomorphism of U onto U/W

B) Attempt any one: 07

c) If F is a field of real numbers and if V is set of all sequence of the form $(a_1, a_2, \dots, a_n, \dots)$, $a_i \in F$.
Let W=$\{(a_1, a_2, \dots, a_n, \dots) \in V | \lim_{n\to\infty} a_n = 0\}$ prove that W is a subspace of V.

d) If W_1, and W_2 are the subspaces of a finite-dimensional vector space V then show that:
A$(W_1 \cap W_2)$ =A $(W_1) + A\ (W_2)$

Q.2 A) Attempt any one: 08

a) If V is finite-dimensional vector space over F and W is a subspace of V, then prove that $\widehat{W}$ is isomorphic to $\widehat{V}$/A(W)

b) If V is finite-dimensional inner product space and if W is a subspace of V, then prove that $V = W + W^{\perp}$

B) Attempt any one: 07

c) In vector space $F^{(n)}$ over F define, for u=($\alpha_1, \alpha_2, \dots, \alpha_n$) and v=($\beta_1, \beta_2, \dots, \beta_n$), (u,v)= $\alpha_1 \overline{\beta_1} + \alpha_2 \overline{\beta_2} + \dots + \alpha_n \overline{\beta_n}$. Show that $F^{(n)}$ is an inner product space.

d) If A and B are submodules of an R-module M, then prove that:
A+B= {a+b|a∈ A, b∈ B } is a submodule of M.

Q.3 A) Attempt any one: 05

a) If u, $v \epsilon$V and $\alpha, \beta\epsilon$F, then prove that $||\alpha u + \beta v||^2 = |\alpha|^2 ||u||^2 + \alpha\overline{\beta}\ (u, v)$+$\bar{\alpha}\beta$ $(v, u) + |\beta|^2\ ||v||^2$

b) If S is non-empty subset of a vector space V, then prove that L(s) is a subspace of V

B) Attempt any one: 05

c) If F is a field of real number ,then prove that the vectors (1,1,0),(3,1,3) and (5,3,3) in $F^{(3)}$ are linearly dependent over F.

d) If V is finite –dimensional inner product space and if $\{w_1, w_2, \ldots, w_m\}$ is an orthonormal set in V such that $\sum_{i=1}^{m}|w_i, v|^2 = \|v\|^2$ for every $v\epsilon\ V$ prove that $\{w_1, w_2, \ldots, w_m\}$ must be a basis of V

Q.4 Choose the correct alternative: 10

i) If dim $V = n$, then the number of vectors in a basis of V is______
a) less than n b) greater than n c) equal to n d) none of these

ii) The dimension of a vector space R^3 over R is_______
a) 2 b) 3 c) 4 d) 1

iii) In an inner product space V, the inequality $\sum_{i=1}^{m}|(w_i, v)|^2 \leq \|v\|^2$ is called______
a) Schwarz inequality b) Bessel's inequality
c)Triangle inequality d) None of these

iv) The norm of the vector (1,-2,5) is _______
a)1 b)4 c) 25 d) $\sqrt{30}$

v) If V is a vector space over F, then elements of V are called_______
a) vectors b) scalars c) constants d) none of these