

**[Time: 1:30 Hours]**

**[Max.Marks:50]**

Please check whether you have got the right question paper.

- N.B
- i) Attempt all questions
  - ii) Figures to the right indicate full marks

- Q.1 (A) Attempt any one : 08
- a) If  $f$  is finitely derivable at  $c$ , then prove that  $f$  is also continuous at  $c$ .

b) If  $y = e^{ax} \cdot \sin(bx+c)$ , then prove that

$$\frac{d^n y}{dx^n} = r^n e^{ax} \sin(bx + c + n\phi),$$

Where  $r = \sqrt{a^2 + b^2}$ ,  $\phi = \tan^{-1} \left( \frac{b}{a} \right)$

- (B) Attempt Any One: 07

c) If  $y = x^2 e^x \cos x$ , then find  $\frac{d^n y}{dx^n}$

d) If  $y = e^{-x} (A \cos x + B \sin x)$ ,

Prove that  $\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 2y = 0$

- Q.2 (A) Attempt any one : 08

a) If a function  $f$  is

i) continuous in a closed interval  $[a, b]$ ,

ii) derivable in the open interval  $(a, b)$

and iii)  $f(a) = f(b)$ , then prove that there exists at least one value  $c \in (a, b)$  such that  $f'(c) = 0$

b) If  $Z = f(x, y)$  be a homogeneous function of  $x, y$  of degree  $n$  then prove that

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = nz, \quad \forall x, y \in \text{the domain of the function}$$

(B) Attempt any one:

07

c) If in the Cauchy's mean value theorem,  $f(x) = e^x$  and  $F(x) = e^{-x}$ , show that  $c$  is the arithmetic mean between  $a$  and  $b$

d) If  $u = \log(x^2 + y^2 + z^2)$ , prove that  $x \frac{\partial^2 u}{\partial y \partial z} = y \frac{\partial^2 u}{\partial z \partial x} = z \frac{\partial^2 u}{\partial x \partial y}$

Q.3 (A) Attempt any one:

05

a) Prove that the gradient of a scalar point function is a vector point function.

b) Prove that

$$\text{div}(\vec{f} \times \vec{g}) = \vec{g} \cdot \text{curl } \vec{f} - \vec{f} \cdot \text{curl } \vec{g}$$

(B) Attempt any one :

05

c) Find  $\text{div } \vec{F}$  and  $\text{curl } \vec{F}$  where

$$\vec{f} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$$

d) Show that  $f(x) = |x|$  is not derivable at 0

Q.4

Choose the correct alternative.

10

i. If  $y = \log(\cosh x)$  then  $\frac{dy}{dx} = \dots$

a)  $\coth x$  b)  $\tanh x$  c)  $\text{sech } x$  d)  $\sinh x$

ii. If  $y = \sin(4x - 5)$  then  $y_3 = \dots$

a)  $4^3 \sin\left(4x - 5 + 3\frac{\pi}{2}\right)$  b)  $4 \cos\left(4x - 5 + \frac{\pi}{2}\right)$   
c)  $\sin\left(4x - 5 + \frac{\pi}{2}\right)$  d)  $4^3 \cos\left(4x - 5 + 3\frac{\pi}{2}\right)$

iii. For every  $x \in R$ ,  $\sin x = \dots$

a)  $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$  b)  $x + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$  c)  $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$  d)  $1 + x + \frac{x^2}{2!} + \dots$

iv.  $\text{Grad}(\phi \psi) = \dots$

a)  $\phi \text{grad } \psi + \psi \text{grad } \phi$  b)  $\text{grad } \phi + \text{grad } \psi$  c)  $\psi \text{grad } \phi - \phi \text{grad } \psi$   
d) none of these

v.  $\text{Curl } \vec{r} = \dots$

a) 1 b) 2 c) 3 d) 0

## SUBJECT CODE NO:- Y-2033

## FACULTY OF SCIENCE

B.Sc. F.Y (Sem-II) Examination March/April 2017

Mathematics MAT - 201 (Integral Calculus)

[Time: 1:30 Hours]

[Max.Marks:50]

Please check whether you have got the right question paper.

N.B

- i) Attempt all questions.  
ii) Figures to right indicate full marks.

Q.1 A) Attempt any one:

a) Obtain a reduction formula for  $\int x^n e^{-x} dx$  and hence show that the improper integral  $\int_0^\infty x^n e^{-x} dx = n!$ , where  $n$  is any positive integer. 08

b) Obtain a reduction formula for  $\int \sin^n x dx$  where  $n$  is positive integer. Hence evaluate  $\int \sin^4 x dx$ . 07

B) Attempt any one:

c) Evaluate:  $\int \frac{(x+7)}{x^2+4x+13} dx$

d) Evaluate:  $\int \frac{(2x-3)}{(x^2-1)(2x+3)} dx$  08

Q.2 A) Attempt any one:

a) Evaluate  $\int_a^b x^2 dx$  as the limit of sum. 08

b) Prove that the area of the region bounded by the curve  $a^4 y^2 = x^5(2a - x)$  is to that of the circle whose radius is  $a$  is 5 to 4. 07

B) Attempt any one:

c) Find the length of the arc of the parabola  $x^2 = 4ay$  measured from the vertex to one extremity of the latus rectum.

d) Find the volume generated by the portion of the arc  $y = \sqrt{1+x^2}$  lying between  $x = 0$  &  $x = 4$ , as it revolves about the axis of  $x$ . 05

Q.3 A) Attempt any one:

a) Prove that the necessary and sufficient condition for a continuous vector point function to be irrotational in a simply connected region  $R$  is that it is the gradient of a scalar point function. 05

b) If  $\vec{F}$  is any continuously differentiable vector point function and  $S$  is a surface bounded by curve  $C$ , then prove that  $\int_C \vec{F} \cdot d\vec{r} = \int_S \text{curl} \vec{F} \cdot \vec{n} ds$ , where the unit normal vector  $\vec{n}$  at any point of  $S$  is drawn in the sense in which a right handed screw would move when rotated in the sense of description of  $C$ . 05

B) Attempt any one:

c) Evaluate  $\int_V (2x + y) dv$ , where  $V$  is closed region bounded by the cylinder  $z = 4 - x^2$  and the plane  $x = 0, y = 0, y = 2$  and  $z = 0$ . 10

d) Evaluate:  $\int_C \vec{F} \cdot d\vec{r}$  where  $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$  and the curve  $C$  is the rectangle in the  $xy$  - plane bounded by  $y = 0, x = a, y = b, x = 0$ .

Q.4 Choose the correct alternatives:

1)  $\int \frac{dx}{(2x-3)^3} = \dots$

a)  $-\frac{1}{4(2x-3)^2}$

b)  $\frac{1}{4(2x-3)^2}$

c)  $\frac{1}{(2x-3)^2}$

d)  $\frac{-1}{(2x-3)^2}$

2)  $\int \sin^3 x \, dx = \dots$

a)  $\cos x + \frac{1}{3} \cos^3 x$

b)  $-\cos x + \frac{1}{3} \cos^3 x$

c)  $\cos x - \frac{1}{3} \cos^3 x$

d)  $-\cos x + \frac{1}{2} \cos^3 x$

3) The length of the arc of the curve  $y = \log \sec x$  between  $x = 0$  &  $x = \frac{\pi}{6}$  is equal to .....

a)  $\log 3$

b)  $2 \log 3$

c)  $\frac{1}{2} \log 3$

d) None of the above.

4) A vector whose divergence is zero is called .....

a) Solenoidal vector

b) Irrotational vector

c) Polar vector

d) Axial vector.

5) Value of  $\oint_C \vec{r} \times d\vec{r} = \dots$

a)  $\iint_S \vec{n} \, ds$

b) Zero

c)  $2 \iint_S \vec{n} \, ds$

d) None of these.

[Time: 1:30 Hours]

[Max.Marks:50]

Please check whether you have got the right question paper.

N.B

Attempt all question

Figures to the right indicate full marks

- Q.1 A) Attempt any one
- a) Explain the method of solving differential equation  $\frac{dy}{dx} + py = Q$ , where P and Q are functions of x or constants 08
- b) Explain the method of solving the differential equation  $\frac{d^n y}{dx^n} + p_1 \frac{d^{n-1} y}{dx^{n-1}} + \dots + p_n y = x$  where  $p_1, p_2, \dots, p_n$  are constants and x is a function of x
- B) Attempt any one 07
- c) Solve  $(x^2 + 1) \frac{dy}{dx} + 2xy = 4x^2$
- d) Solve  $\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = e^{4x}$
- Q.2 A) Attempt any one
- a) Explain the method of solving the differential equation 08
- $(a + bx)^n \frac{d^n y}{dx^n} + p_1 (a + bx)^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots + p_{n-1} (a + bx) \frac{dy}{dx} + p_n y = f(x)$ , where  $p_1, p_2, \dots, p_n$  are constants
- b) Solve  $x^2 \frac{d^2 y}{dx^2} + 4x \frac{dy}{dx} + 2y = e^x$  07
- B) Attempt any one
- c) Solve  $x^3 \frac{d^3 y}{dx^3} + 3x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$
- d) Solve  $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = 2 \log x$
- Q.3 A) Attempt any one 05
- a) Explain the method of solving equation  $\frac{d^n y}{dx^n} = f(x)$
- b) Explain the method of solving simultaneous differential equations  $\frac{dx}{p} = \frac{dy}{Q} = \frac{dz}{R}$ , where P,Q,R are functions of x,y,z.
- B) Attempt any one 05
- c) Solve  $\cos^2 x \frac{dy}{dx} + y = \tan x$
- d) Form the partial differential equation from the equation  $Z = a(x+y) + b$

Q.4 Choose correct alternative

10

1) The necessary and sufficient condition for differential equation  $Mdx+Ndy=0$  is to be exact is \_\_\_\_\_

a)  $\frac{\partial M}{\partial Y} = \frac{-\partial N}{\partial x}$

b)  $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

c)  $\frac{\partial M}{\partial Y} = \frac{\partial N}{\partial x}$

d)  $\frac{\partial M}{\partial x} = \frac{-\partial N}{\partial Y}$

2) The integrating factor of the differential equation  $\frac{dy}{dx} + y = e^{-x}$  is \_\_\_\_\_

- a) X
- b)  $\cos x$
- c)  $-x$
- d)  $e^x$

3) The particular integral of differential equation  $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 2e^{2x}$  is \_\_\_\_\_

- a)  $\frac{2}{5} e^{2x}$
- b)  $\frac{1}{5} e^{2x}$
- c)  $2e^{2x}$
- d)  $e^{2x}$

4) The partial differential equation formed by eliminating constants a and b from  $z=ax+by+ab$  is \_\_\_\_\_

- a)  $Z=px+qy+pq$
- b)  $Z= pq$
- c)  $Pq=1$
- d) None of these

5) The partial differential equation corresponds to \_\_\_\_\_

- a) Single independent variable
- b) More than one independent variable
- c) Single ordinary derivative
- d) Note of these

## SUBJECT CODE NO:- Y-2034

## FACULTY OF SCIENCE

## B.Sc. F.Y (Sem-II) Examination March/April 2017

## Mathematics MAT - 202

## (Geometry)

[Time: 1:30 Hours]

[Max.Marks:50]

Please check whether you have got the right question paper.

N.B

- i) Attempt all questions.  
ii) Figures to right indicate full marks.

Q.1A) Attempt any one:

08

a) Prove that every equation of the first degree in  $x, y, z$  represent a plane.b) Find the equation of the line passing through a given point  $A(x, y, z)$  and having direction cosines  $l, m, n$ .

B) Attempt any one:

07

c) Find the equation of the plane through the three points  $(1,1,1), (1,-1,1), (-7,-3,-5)$  and show that it is perpendicular to the XZ-plane.

d) Find the magnitude and the equations of the line of shortest distance between the lines:

$$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}, \quad \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}.$$

Q.2A) Attempt any one:

08

a) Find the condition that the two given straight lines,

$$\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}, \quad \frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$$

are co-planer.

b) Prove that a plane section of a sphere is a circle.

B) Attempt any one :

07

c) Find the length of the perpendicular from the point  $(4,-5,3)$  to the line  $\frac{x-5}{3} = \frac{y+2}{-4} = \frac{z-6}{5}$ .d) Find the co-ordinates of the points where the line  $\frac{x+3}{4} = \frac{y+4}{3} = \frac{-(z-8)}{5}$  intersects the sphere

$$x^2 + y^2 + z^2 + 2x - 10y = 23.$$

Q.3A) Attempt any one :

05

a) Define right circular cone and show that every section of a right circular cone by a plane perpendicular to its axis is a circle.

b) Find the points of intersection of the line  $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$  with the central conicoid  $ax^2 + by^2 + cz^2 = 1$ 

B) Attempt any one :

05

c) Find the equation of the right circular cylinder of radius 2 whose axis is the line  $\frac{(x-1)}{2} = \frac{(y-2)}{1} = \frac{(z-3)}{2}$ .d) Find the equations to the tangent planes to  $7x^2 - 3y^2 - z^2 + 21 = 0$  which pass through the line  $7x - 6y + 9 = 0, z = 3$ .

Q.4 Choose the correct alternatives :

1) The equation to a plane in normal form is .....

a)  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

b)  $\frac{x}{l} + \frac{y}{m} + \frac{z}{n} = 1$

c)  $ax + by + cz = P$

d)  $lx + my + nz = P$ .

2) The number of arbitrary constants in the equations of a straight line is .....

a) 6

b) 4

c) 2

d) 0.

3) The radius of the sphere  $x^2 + y^2 + z^2 - 6x + 8y - 10z + 1 = 0$  is .....

a) 7

b)  $\frac{1}{7}$

c) 49

d) 1.

4) Equation to the right circular cone whose vertex is at origin, the axis along x-axis and semi-vertical angle  $\alpha$ , is .....

a)  $x^2 + y^2 = z^2 \tan^2 \alpha$

b)  $y^2 + z^2 = x^2 \tan^2 \alpha$

c)  $y^2 \tan^2 \alpha$

d) None of these.

5) The condition that the plane  $lx + my + nz = P$  may touch the conicoid  $ax^2 + by^2 + cz^2 = 1$  is .....

a)  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} = P$

b)  $\frac{l^2}{a} + \frac{m^2}{b} + \frac{n^2}{c} = P^2$

c)  $\frac{l}{a^2} + \frac{m}{b^2} + \frac{n}{c^2} = P^2$

d)  $\frac{l^2}{a} + \frac{m^2}{b} + \frac{n^2}{c} = P$ .



SUBJECT CODE NO:- Y-2041

FACULTY OF SCIENCE

B.Sc. S.Y (Sem-IV) Examination March/April 2017

Mathematics MAT - 402 (Revised)

Partial Differential Equation

[Time:1:30Hours]

[Max.Marks:50]

Please check whether you have got the right question paper.

- N.B
- All questions are compulsory.
  - All questions carry equal marks.
  - Figures to the right indicate full marks.
  - Attempt either part (A) or part (B) for questions 1 to 4.

- Q.1A) a) Show that  $\frac{dx}{p} = \frac{dy}{q} = \frac{dz}{R}$  are the auxiliary equations for the partial differential equation  $Pp + Qq = R$ . 05  
 b) Solve: 05  
 $p\sqrt{x} + q\sqrt{y} = \sqrt{z}$
- OR**
- B) c) Define complete integral of the partial differential equation. Find the complete integral of the partial differential equation of the type  $f_1(x, p) = f_2(y, q)$  05  
 d) Find the complete integral of: 05  
 $p^3 + q^3 = 27z$
- Q.2A) a) Explain the Charpit's method to solve the partial differential equation. 05  
 b) Solve: 05  
 $z = p(x + p) + q(y + q)$
- OR**
- B) c) Explain Jacobi's method for solving the partial differential equations. 05  
 d) Solve:- 05  
 $p_1^3 + p_2^2 + p_3 = 1$
- Q.3A) a) Write the working rule to find complementary function of the equation. 05  
 $(A_0 D^n + A_1 D^{n-1} D' + \dots + A_n D'^n)Z = 0$   
 b) Find the particular integral of: 05  
 $(D^2 - 2DD' + D'^2)Z = 12xy$
- OR**
- B) c) Explain the method of finding particular integral of non-homogeneous partial differential equation 05  
 $F(D, D')Z = e^{ax+by}$   
 d) Solve: 05  
 $(D^2 - DD' - 2D'^2)z = (y - 1)e^x$
- Q.4A) a) Obtain the Monge's subsidiary equations for the equation  $Rr + Ss + Tt = V$ , where R, S, T and V are function of x, y, z, p and q. 05  
 b) Obtain Monge's subsidiary equations for the equation  $r = a^2 t$  05
- OR**

- B) c) Obtain the canonical form of the equation.

$$A \frac{\partial^2 z}{\partial x^2} + 2B \frac{\partial^2 z}{\partial x \partial y} + C \frac{\partial^2 z}{\partial y^2} F \left( x, y, z, \frac{\partial z}{\partial u}, \frac{\partial z}{\partial v} \right) = 0$$

Where  $S^2 - 4RT < 0$

- d) Solve:

$$2p_1x_1x_2 + 3p_2x_3^2 + p_2^2p_3 = 0$$

Q.5 Choose the correct alternative and fill the blanks:

- The complementary function is the general solution of the equation -----  
 a)  $F(D, D')z = 0$   
 b)  $F(D, D')z = f(x, y)$   
 c)  $F(D)z = 0$   
 d) None of the above
- If  $Pp + Qq = R$  then the direction ratios of the normal at a point on the surface  $f(x, y, z) = 0$  are -----  
 a)  $p, q, 1$   
 b)  $-p, -q, 1$   
 c)  $p, -q, 1$   
 d)  $p, q, -1$
- The complete integral of the equation  $z = px + qy + pq$  is given by -----  
 a)  $z = ax + by + ab$   
 b)  $z = ax + by + a + b$   
 c)  $z = ax + by$   
 d) None of the above
- If  $\frac{y^2z}{x}p + zxq = y^2$ , then Lagrange's auxiliary equations are -----  
 a)  $\frac{dx}{\frac{y^2z}{x}} = \frac{dy}{zx} = \frac{dz}{y^2}$   
 b)  $\frac{dx}{zx} = \frac{dy}{y^2} = \frac{dz}{\frac{y^2z}{x}}$   
 c)  $\frac{dx}{\frac{y^2z}{x}} = \frac{dy}{zx} = \frac{dz}{y^2}$   
 d)  $\frac{dx}{zx} = \frac{dy}{y^2} = \frac{dz}{y^2}$
- The equation  $p^2 + q^2 = n^2$  is of the standard form -----  
 a)  $f(p, q) = 0$   
 b)  $f(z, p, q) = 0$   
 c)  $f(x, p) = g(y, q)$   
 d) none of the above

**SUBJECT CODE NO:- Y-2201**  
**FACULTY OF SCIENCE**  
**B.Sc. S.Y (Sem-III) Examination March/April 2017**  
**Mathematics MAT - 302 (Revised)**  
**Integral Transforms**

[Time:1:30Hours]

[Max.Marks:50]

Please check whether you have got the right question paper.

N.B

- i) Attempt all questions  
 ii) Figures to the right indicate full marks.

Q.1 A) Attempt any one :

08

a) Prove that

$$\Gamma(l, m) = \frac{\Gamma(l) \Gamma(m)}{\Gamma(l+m)}, \text{ where } l, m \text{ are integer}$$

b) Prove that

$$\Gamma(m) \Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma(2m), \text{ where } m \text{ is an integer.}$$

B) Attempt any one :

07

c) Show that

$$\Gamma\left(\frac{1}{n}\right), \Gamma\left(\frac{2}{n}\right), \Gamma\left(\frac{3}{n}\right), \dots, \Gamma\left(\frac{n-1}{n}\right) = \frac{(2\pi)^{\frac{n-1}{2}}}{n^{1/2}}, \text{ where } n \text{ is an integer}$$

d) Find Laplace transform of  $\int_0^t \left(\frac{1-e^{-2x}}{x}\right) dx$ .

Q.2 A) Attempt any one :

08

a) If  $L\{F(t)\} = f(s)$ , then prove that

$$L\{t^n F(t)\} = (-1)^n \frac{d^n f(s)}{ds^n}, \text{ for } n = 1, 2, 3, \dots$$

b) If  $L^{-1}\{f(s)\} = F(t)$ ,  $L^{-1}\{g(s)\} = G(t)$ , then prove that

$$L^{-1}\{f(s).g(s)\} = \int_0^t F(u) G(t-u) du$$

B) Attempt any one :

07

c) Show that

$$L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\} = \frac{1}{2a^3} \{ \sin at - at \cos at \}$$

d) Find

$$L^{-1}\left\{\frac{e^{\left(\frac{1}{s}\right)}}{s}\right\}$$

Q.3 A) Attempt any one :

05

a) If  $F(t) = \begin{cases} e^{-xt} \Phi(t), & t > 0 \\ 0, & t < 0 \end{cases}$   
 then prove that  $F\{F(t)\} = L\{\Phi(t)\}$

b) If  $F\{F(x)\} = f(s)$  then prove that  $F\{F(x) \cos ax\} = \frac{1}{2} f(s-a) + \frac{1}{2} f(s+a)$

B) Attempt any one :

c) Find the Fourier transform of

$$f(x) = \begin{cases} x, & |x| \leq a \\ 0, & |x| > a \end{cases}$$

d) Solve

$$(D^3 - D^2 + 4D - 4)y = 68e^t \sin 2t,$$

if  $y = 1$ ,  $Dy = -19$ ,  $D^2y = -37$  at  $t = 0$  using Laplace transform.

05

Q.4 Choose the correct alternative :

10

1] The value of the integral  $\int_0^1 x^4(1-x)^3 dx$  is

- a)  $\frac{1}{280}$
- b)  $\frac{2}{289}$
- c) 280
- d) 2

2] The value of  $\Gamma(\frac{1}{2})$  is

- a)  $\pi$
- b)  $\pi^2$
- c)  $\sqrt{\pi}$
- d)  $2\pi$

3] The Laplace transform of the function  $f(t) = 1$  is .....

- a)  $\frac{1}{s}, S > 0$
- b)  $\frac{1}{s+1}, S > 0$
- c)  $S, S > 0$
- d)  $\frac{1}{s^2}, S > 0$

4]  $L\{\cos h at\} = \frac{a}{s^2 - a^2}$

- a)  $\frac{a}{s^2 - a^2}$
- b)  $\frac{s^2 - a^2}{s}$
- c)  $\frac{s^2 + a^2}{s}$
- d)  $\frac{s^2 + a^2}{s^2 - a^2}$

5] The Fourier sine transform of  $F(x) = \frac{1}{x}$  is.....

- a)  $\sqrt{\pi}$
- b)  $\sqrt{\frac{2}{\pi}}$
- c)  $\sqrt{2\pi}$
- d)  $\sqrt{3\pi}$

**SUBJECT CODE NO:- Y-2039**  
**FACULTY OF SCIENCE**  
**B.Sc. S.Y (Sem-IV) Examination March/April 2017**  
**Mathematics MAT - 401 (Revised)**  
**Numerical Methods**

[Time: 1:30 Hours]

[Max.Marks:50]

Please check whether you have got the right question paper.

N.B

- i) All questions are compulsory.
- ii) Figures to the right indicate full marks.
- iii) Attempt either part (A) or part (B) for questions 1 to 4.

- Q.1 (A) (a) Explain the method of Newton-Raphson for obtaining root of an Equation  $f(x)=0$ . 05  
 (b) Find the values of a,b and c so that  $y=a+bx+cx^2$  is the best fit to the data: 05

X	0	1.0	2.0
Y	1.0	6.0	17.0

OR

- (B) (c) Show that:- 05  
 $T_{n+1}(x)=2xT_n(x)-T_{n-1}(x)$  in Chebyshev Polynomial.

- (d) Find a real root of the equation  $x^3+x^2+x+7=0$ , using bisection method. 05

- Q.2 (A) (a) Show that:- 05  

$$e^x(u_0+x\Delta u_0+\frac{x^2}{2!}\Delta^2 u_0+\dots)=u_0+u_1x+u_2\frac{x^2}{2!}+\dots$$
 05  
 (b) Determine Hermite polynomial of degree five which fits the following data. 05

X	2	2.5	3.0
$y=\ln x$	0.69315	0.91629	1.09861
$y'=\frac{1}{x}$	0.5	0.4	0.3333

OR

- (B) (c) Derive Newton's forward interpolation formula for equal intervals. 05  
 (d) Find a polynomial in x, using the following table: 05

x	-1	0	3	6	7
f(x)	3	-6	39	822	1611

- Q.3 (A) (a) Explain the method of solving linear equations using iterative method. 05  
 (b) Using Picard's method obtain the solution of  $y'=x(1+x^3y)$ ,  $y(0)=3$ . 05

OR

- (B) (c) Explain the method of obtaining the solution of  $y'=f(x,y)$ ,  $y(x_0)=y_0$ , using Taylor's series. 05  
 (d) Solve the equations:- 05

$$2x+y+z=10$$

$$3x+2y+3z=18$$

$$x+4y+9z=16$$

By Gauss elimination method.

Q.4 (A) (a) Prove that  $T_n x$  is a polynomial of degree  $n$  in  $x$ .

05

(b) Show that:-

05

$$\Delta^n u_{x-n} = u_x - n u_{x-1} + \frac{n(n-1)}{2} u_{x-2} + \dots + (-1)^n u_{x-n}.$$

OR

05

(B)

(c) Define:-

(i) Eigen value

(ii) Eigen vector

(iii) Characteristic equation.

05

(d) Solve  $\frac{dy}{dx} = 1 + y^2$ , where  $y=0$ , when  $x=0$ . Find  $y(0.2), y(0.4)$  using Runge-Kutta method.

Q.5 Choose the correct alternative and fill the blanks-

10

(i)  $\Delta^3 y_0 = \dots$

(a)  $y_3 + 2y_2 - 3y_1 + y_0$

(b)  $y_3 - 3y_2 + 2y_1 - y_0$

(c)  $y_3 - 2y_2 + 2y_1 - y_0$

(d)  $y_3 - 2y_2 + 2y_1 + y_0$

(ii) If  $y = a_0 + a_1 x + a_2 x^2$  then the third normal equation by least square method is  $\sum x_i^2 y_i = \dots$

(a)  $na_0 + \sum a_1 x_i + a_2 \sum x_i^2$

(b)  $a_0 \sum x_i^2 + a_1 \sum x_i^3 + a_2 \sum x_i^4$

(c)  $a_0 \sum x_i + a_1 \sum x_i^2 + a_2 \sum x_i^3$

(d)  $a_0 \sum x_i^2 + a_1 \sum x_i^4 + a_2 \sum x_i^6$

(iii) If a matrix  $A$  is factorized into the form  $LU$  where  $L$  is -----.

(a) Unit matrix

(b) Unit upper triangular matrix

(c) Unit lower triangular matrix

(d) Zero matrix

(iv) If  $y' = y - x$  and  $y(0) = 2, h = 0.2$  then by Euler's method the value of  $y_1 = \dots$

(a) 2.02

(b) 2.2

(c) 2.4

(d) 2.04

(v)  $\Delta(e^x) = \dots$  taking  $h=1$ .

(a)  $(e-1)e^x$

(b)  $(e+1)e^x$

(c)  $ce^x$

(d)  $(1+e)e^x$

**SUBJECT CODE NO:- Y-2238**  
**FACULTY OF SCIENCE**  
**B.Sc. S.Y (Sem-III) Examination March/April 2017**  
**Mathematics MAT - 303**  
**Mechanics-I**

[Time : 1:30 hours]

[Max.Marks:50]

Please check whether you have got the right question paper.

- N.B
- i. Attempt all questions
  - ii. Figures to the right indicate full marks.

Q.1A) Attempt any one 08

- a) Prove that the algebraic sum of the resolved parts of two forces in a given direction is equal to the resolved part of their resultant along the same direction.
- b) Find the magnitude and direction of the resultant of any number of the coplanar forces acting at a point.

B) Attempt any one 07

- a) A body of weight 52kg suspended by two strings of lengths 5m and 12m attached to the point s in the same horizontal line whose distance apart is 13m. Find the tensions of the strings.
- b) Three forces of magnitudes P,Q,R acting on a particle are in equilibrium and the angle between P and Q is double the angle between P and R. show that  $R^2 = Q(Q - P)$

Q.2A) Attempt any one 08

- a) Prove that the sum of the vector moments of two like parallel forces acting on a rigid body about any point equals to the vector moment of their resultant about same point.
- b) Define couple. Show that the magnitude of moment of the couple equals to the product of magnitudes of a force in the couple and arm of the couple.

B) Attempt any one 07

- a) A force  $\vec{F}$  of magnitudes 8 units acts at a point P(2, 3, 4) along the line  $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ . Find the moment of the force  $\vec{F}$  about X-axis.
- b) Define Centre of graving of the rigid body. Determine the Centre of gravity of uniform parallelogram.

Q.3A) Attempt any one 05

- a) Define
  - i. Moment of a force
  - ii. Resolved part of a force
- b) Three forces of magnitudes P ,Q ,R act along the sides BC, CA, AB, of a  $\Delta ABC$  taken in order, prove that if the resultant passess through the circumcentre of  $\Delta ABC$ , then  

$$P \cos A + Q. \cos B + R. \cos C = 0$$

B) Attempt any one 05

- c) Define centroid of the weighted points and show that it is unique.
- d) Two forces whose magnitudes are P and  $P\sqrt{2}$  act on a particle in the direction inclined at an angle

135° to each other. Find the magnitude and direction of the resultant.

Q.4 Choose the correct alternative

10

- 1) Statics is a part of Mechanics which deal with the-----.
  - a) Equilibrium of system in motion
  - b) Equilibrium of system in rest
  - c) Both a and b
  - d) None of these
- 2) Two forces acting at two different point of a rigid body are said to form a couple if they are -----
  - a) Equal
  - b) Unlike
  - c) Parallel
  - d) All the above
- 3) If TWO forces of magnitudes 8kg and 6kg act at right angles then the magnitudes of their resultant force is-----.
  - a) 20kg
  - b) 5kg
  - c) 15kg
  - d) 10kg
- 4) The effect of the couple acting on the body produces -----
  - a) Only a motion of rotation
  - b) Only a motion of translation
  - c) Both a and b
  - d) None of these
- 5) Centre of gravity of rigid body is-----.
  - a) Unique
  - b) Non – unique
  - c) A line
  - d) None of these



**SUBJECT CODE NO:- Y-2079**  
**FACULTY OF SCIENCE**  
**B.Sc. S.Y (Sem-IV) Examination March/April 2017**  
**Mathematics MAT - 403 (Revised)**  
**Mechanics-II**

**[Time:1:30Hours]**

**[Max.Marks:50]**

Please check whether you have got the right question paper.

- N.B
- i) Attempt all questions.
  - ii) Figures to the right indicate full marks.
  - iii) Draw well labelled diagram whenever necessary.

Q.1A) Attempt any one: 08

- a) Find the expression for the velocity and acceleration in terms of vector derivatives.
- b) Find the radial and transverse component of acceleration.

B) Attempt any one: 07

- c) A particle moves along a curve  $\Omega = a(1 + \cos\theta)$  with uniform speed  $\vartheta$ . Show that.

$$\frac{d\theta}{dt} = \frac{\vartheta \sec(\theta/2)}{2a} = \frac{\vartheta}{\sqrt{2ar}} \text{ and the radial component of the acceleration is constant.}$$

- d) Prove that the areal velocity  $p$  is  $\frac{1}{2} \vec{\Omega} \times \vec{\vartheta}$ .

Q.2A) Attempt any one: 08

- a) Prove that the kinetic energy of particle of mass  $m$  moving with velocity  $\vec{V}$  is  $mV^2$ . Also prove that the change in kinetic energy of the particle is equal to the work done.
- b) Find the vertex and the latus rectum of the parabola.

B) Attempt any one: 07

- c) A shell burst on striking a ground and its pieces fly in all directions, with maximum speed  $\vartheta$ . Find the time for which a person at a distance  $a$ , is in danger.
- d) A particle is projected at an angle  $\alpha$  to the horizon with speed  $\mu$ . If  $R$  is the horizontal range, prove that its path can be put in the form.  $y = x \tan \alpha (1 - \frac{x}{R})$ .

Q.3A) Attempt any one: 05

- a) Find the differential equation of central orbit in pedal form.
- b) A particle moves in an ellipse under a central force directed towards its focus, whose polar equations is  $\frac{l}{r} = 1 + e \cos\theta$ , focus being the pole, find the law of force and the velocity at any point of its path.

B) Attempt any one: 05

- c) Find the law of force under which the curve  $r = a \cos\theta$ .
- d) A boy sitting on the top of a tower 96 ft high throws a stone with speed 80 ft/sec at an elevation of  $30^\circ$  to the horizon. Find the time the stone to reach the horizontal plane through the foot of the tower.

Q.4 Choose the correct alternative and rewrite the sentence:

10

- 1) The unit of angular acceleration is.....
  - a) rad/sec
  - b)  $\text{rad/sec}^2$
  - c) m/sec
  - d) rad.
- 2) The acceleration of a point moving in a plane curve with uniform speed is given by.....
  - a)  $\rho \left(\frac{d\psi}{dt}\right)^2$
  - b)  $\rho \left(\frac{d\psi}{dt}\right)$
  - c)  $\left(\frac{d\psi}{dt}\right)$
  - d) None of these.
- 3) The work done by the force  $\vec{F} = 2x\vec{i} + 2y\vec{j}$  in moving particle from P(1, 2) to Q(3, 2) is .....
  - a) 13
  - b) 5
  - c) 8
  - d) 26.
- 4) If the force is acting towards a fixed point then it is called.....
  - a) Central repulsive force
  - b) Tangential force
  - c) Terminal force
  - d) Central attractive force.
- 5) The line joining the centre of force and apse point is called.....
  - a) A straight line
  - b) A skew line
  - c) An apse line
  - d) A dotted line.

**SUBJECT CODE NO:- Y-2199**  
**FACULTY OF SCIENCE**  
**B.Sc. S.Y (Sem-III) Examination March/April 2017**  
**Mathematics MAT -301 (Revised)**  
**Number Theory**

**[Time:1:30Hours]**

**[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B

- i) Attempt all questions.
- ii) Figure to the right indicates full Marks.

- Q.1 A) Attempt any one. 8
- a) If  $a$  and  $b$  integers, both of which are not zero, Then prove that, Then prove that, There exist integers  $x$  and  $y$  such that  $\gcd(a,b)=ax+by$ .
  - b) Prove that the linear Diophantine equation  $ax+by=c$  has a solution if and only if  $d \mid c$ , where  $d=\gcd(a,b)$ .
- B) Attempt any one. 7
- c) Solve the Diophantine equation.  
 $172x+20y=1000$
  - d) Find  $\text{lcm}(3054, 12378)$ .
- Q.2 A) Attempt any one 8
- a) If  $ca \equiv cb \pmod{h}$ , then prove that  $a \equiv b \pmod{\frac{h}{d}}$ , where  $d=\gcd(c,h)$
  - b) If  $p$  is a prime and  $p \nmid a$  then show that  $a^{p-1} \equiv 1 \pmod{p}$ .
- B) Attempt any one 7
- c) Show that 41 divides  $2^{40}-1$ .
  - d) Solve the Congruence  
 $x \equiv 5 \pmod{6}, x \equiv 4 \pmod{11}, x \equiv 3 \pmod{17}$ .
- Q.3 A) Attempt any one 5
- a) Prove that the function  $\mu$  is a multiplication Function.
  - b) If the integer  $n > 1$  has the prime factorization,  $n = p_1^{k_1} p_2^{k_2} \dots p_n^{k_n}$  then prove that  $\Phi(n) = n(1 - \frac{1}{p_1})(1 - \frac{1}{p_2}) \dots (1 - \frac{1}{p_n})$

B) Attempt any one

c) Calculate  $\Phi(5040)$ .

d) Calculate  $\tau(180)$

5

Q.4 Choose the correct alternative.

10

1) For any two positive integers a and b. if  $gc(a,b)=6$  and  $lcm(a,b)=60$  then

a)  $ab=90$    b)  $ab=60$    c)  $ab=180$    d)  $ab=360$

2) If  $a=119$  and  $b=272$  then  $gcd(a,b)$  is

a) 17   b) 19   c) 13   d) 11

3) Which Congruence is not correct from

a)  $3 \equiv 24 \pmod{7}$

b)  $-31 \equiv 11 \pmod{7}$

c)  $-15 \equiv -64 \pmod{7}$

d)  $25 \equiv 12 \pmod{7}$

4) If p is prime number and  $p \nmid a$  then

a)  $a^{p-1} \equiv 0 \pmod{p}$

b)  $a^{p-1} \equiv p \pmod{p}$

c)  $a^{p-1} \equiv 1 \pmod{p}$

d) None of these.

5) If  $n=180$  then

a)  $\tau(n)=12$

b)  $\tau(n)=18$

c)  $\tau(n)=14$

d)  $\tau(n)=16$

**SUBJECT CODE NO:- Y-2205****FACULTY OF SCIENCE****B.Sc. T.Y (Sem-V) Examination March/April 2017****Mathematics MAT-503 OR 504 OR 505 (Revised)****1) Mathematical Statistics -I****[Time: 1:30 Hours]****[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B

- i) All questions are compulsory.  
 ii) Figures to the right indicate full marks.  
 iii) Calculator is allowed.

Q.1

A) Attempt any one:

- a) Prove that the sum of the squares of the deviations of a set of values is minimum when taken about mean. 08  
 b) Explain frequency polygon with suitable example. 08

B) Attempt any one:

c) Find the geometric mean for the following data:

85,70,15,75,500,8,45,250,40,36. 07

d) Find the mean of the following frequency distribution: 07

Class intervals	20-30	30-40	40-50	50-60	60-70
Frequency	3	5	20	10	5

Q.2

A) Attempt any one:

- a) Prove that variance is independent of change of origin. 08  
 b) What is standard deviation? Explain its superiority over other measures of dispersion. 08

B) Attempt any one:

c) A distribution consists of three components with frequencies 200,250 and 300 having means 25,10 and 15 and standard deviations 3,4, and 5 respectively. 07

Find the mean and standard deviation of the combined group.

d) If two dice are thrown, what is the probability that the sum is greater than five. 07

Q.3

A) Attempt any one:

a) If  $B \subset A$ , then show that: 05

i)  $P(A \cap B) = P(A) - P(B)$ ,

ii)  $P(B) \leq P(A)$

b) Define: 05

i) Geometric mean

ii) Harmonic mean

iii) Weighted mean

B) Attempt any one:

c) Find the mean deviation about mean of the following distribution: 05

x: 3 4 5

f: 2 3 2

d) Find the value of the constant k such that 05

$f(x) = \begin{cases} kx(1-x), & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$  is a probability density function.

Q.4 For the questions given below. Choose the correct answer.

10

i) The algebraic sum of the deviations of all the variate values from their arithmetic mean is

a) 10   b) 100   c) 1   d) zero

ii) The point of intersection of the 'less than' and the 'greater than' ogive corresponds to

a) mean   b) median   c) mode   g) geometric mean

iii) The skewness of the distribution 4, 4, 5, 5 is

a) 2   b) 4   c) 5   d) zero

iv) If  $P(A) = 0.37$ ,  $P(B) = 0.48$  and  $P(A \cup B) = 0.85$  then  $P(A \cap B)$  is equal to

a) 3.7   b) 8.5   c) zero   d) 38

v) The square of standard deviation is

a) mean   b) mean deviation   c) variance   d) median deviation

**SUBJECT CODE NO:- Y-2205**  
**FACULTY OF SCIENCE**  
**B.Sc. T.Y (Sem-V) Examination March/April 2017**  
**Mathematics MAT-503 OR 504 OR 505 (Revised)**  
**2) Ordinary Differential Equation -I**

[Time: 1:30 Hours]

[Max.Marks:50]

- N.B Please check whether you have got the right question paper.
- i) All questions are compulsory.
  - ii) Figures to the right indicate full marks.

Q.1 A Attempt any one:

(a) If  $p$  is a polynomial of degree  $n \geq 1$ , with leading coefficient 1 (the coefficient of  $z^n$ ), and if  $r$  is a root of  $p$ .  
 Then Prove that: 08

$$P(z) = (z-r)q(z)$$

Where  $q$  is a polynomial of degree  $n-1$ , with leading coefficient 1.

(b) Consider the equation

$$y' + ay = b(x), \text{ where } a \text{ is a constant, } b \text{ is a continuous functions on an interval } I.$$

If  $x_0$  is a point in  $I$  and  $c$  is any constant. Then prove that the function  $\varphi$  defined by 08

$$\varphi(x) = e^{-ax} \int_{x_0}^x e^{at} b(t) dt + c e^{-ax} \text{ is solution of this equation. Also prove every solution has this form.}$$

B Attempt any one:

(c) Consider the equation

$$Ly' + Ry = E \text{ where } L, R, E \text{ are positive constants}$$

i) Solve this equation.

ii) Find the solution  $\varphi$  satisfying  $\varphi(0) = I_0$ , where  $I_0$  is given positive constant.

iii) Show that every solution tends to  $E/R$  as  $x \rightarrow \infty$ .

(d) Consider the equation 07

$$y' + 5y = 2$$

a) Show that the function  $\varphi$  given by

$$\varphi(x) = \frac{2}{5} + ce^{-5x} \text{ is solution, where } c \text{ is any constant.}$$

b) Assuming every solution has this form, find the solution satisfying  $\varphi(1) = 2$

Q.2 A Attempt any one:

a) Prove that for any real  $x_0$ , and constants  $\alpha, \beta$  there exists a solution  $\varphi$  of the initial value problem: 08

$$L(y) = y'' + a_1 y' + a_2 y = 0$$

$$y(x_0) = \alpha, y'(x_0) = \beta \text{ on } -\infty < x < \infty$$

b) Suppose  $\varphi_1, \varphi_2$  are any two linearly independent solutions of 08

$$L(y) = y'' + a_1 y' + a_2 y = 0 \text{ on an interval } I. \text{ Prove that every solution } \varphi \text{ of } L(y) = 0 \text{ Can be written uniquely as}$$

$$\varphi = c_1 \varphi_1 + c_2 \varphi_2 \text{ where } c_1 \text{ and } c_2 \text{ are constants.}$$

B Attempt any one:

c) Find all solutions of

$$y'' + 4y = \cos x$$

b) Find the solutions of the following initial value problem:

$$y'' - 2y' - 3y = 0$$

$$y(0) = 0, y'(0) = 1$$

Q.3 A) Attempt any one:

a) Prove that two solutions  $\varphi_1, \varphi_2$  of

$$L(y) = y'' + a_1 y' + a_2 y = 0$$

Are linearly independent if

$$W(\varphi_1, \varphi_2)(x) \neq 0.$$

b) Prove that for all real  $\theta$

$$e^{i\theta} = \cos \theta + i \sin \theta$$

B) Attempt any one:

c) Find all solutions of

$$y' + 2xy = x$$

d) Show that the functions  $\varphi_1, \varphi_2$  defined by

$$\varphi_1(x) = \cos x$$

$$\varphi_2(x) = 3(e^{ix} + e^{-ix}) \quad \text{are linearly independent for } -\infty < x < \infty$$

Q.4 Choose the correct alternative:

i) The problem  $y' = f(x, y)$  with  $y(x_0) = x_0$  is called -----

a) Initial value problem

b) Boundary value problem

c) both a) & b)

d) None of these

ii) If  $p$  is a polynomial,  $\deg p = n > 1$ , with leading coefficient  $a_0 \neq 0$ . Then  $p$  has exactly -----

a)  $(n-1)$  roots

b)  $n$  roots

c)  $n+1$  roots

d) None of these

iii) Solution of the equation  $y' + ay = 0$  is -----

a)  $\varphi(x) = ce^{ax}$

b)  $\varphi(x) = -ce^{ax}$

c)  $\varphi(x) = ce^{-ax}$

d) None of these

iv) If  $\varphi_1, \varphi_2$  are two solutions of

$$L(y) = y'' + a_1 y' + a_2 y = 0 \text{ on an interval } I \text{ containing a point } x_0 \text{ then } W(\varphi_1, \varphi_2)(x) = \text{-----}$$

a)  $e^{a_1(x-x_0)} W(\varphi_1, \varphi_2)(x_0)$

b)  $e^{-a_1(x-x_0)}$

c)  $e^{-a_1(x-x_0)} W(\varphi_1, \varphi_2)(x_0)$

d) None of these

v) Let  $b$  be continuous on an Interval  $I$ . Every solution  $\Psi$  of  $L(y) = y'' + a_1 y' + a_2 y = b(x)$  on an interval  $I$  can be written as  $\Psi = \Psi_p + c_1 \varphi_1 + c_2 \varphi_2$ , then  $\Psi_p$  is known as -----

a) Complementary functions

b) Particular Solution

c) Complete solution

d) None of these.



Total No. of Printed Pages:1

**SUBJECT CODE NO:- Y-2205**  
**FACULTY OF SCIENCE**  
**B.Sc. T.Y (Sem-V) Examination March/April 2017**  
**Mathematics MAT-503 OR 504 OR 505 (Revised)**  
**3) Programming in C -I**

**[Time: 1:30 Hours]**

**[Max.Marks:40]**

N.B Please check whether you have got the right question paper.

- i) All questions are compulsory.
- ii) Figures to the right indicate full marks.
- iii) Assume the data wherever not given with justification.

- |     |  |    |
|-----|--|----|
| Q.1 | A) Attempt any one :<br>a) Explain printf function in C program with example.<br>b) Explain trigraph characters in C program.  | 05 |
|     | B) Attempt any one:<br>c) Write a program in C using a user-defined function.<br>d) Write a C program for storage classes.   | 05 |
| Q.2 | A) Attempt any one:<br>a) Discuss real constants in C language with example.<br>b) Discuss character set in C language   | 05 |
|     | B) Attempt any one:<br>c) Write a program to calculate the average of a set of 10 numbers.<br>d) Write a program to print a sequence of squares of numbers using shorthand operator *+=. | 05 |
| Q.3 | A) Attempt any One:<br>a) Explain arithmetic operators and integer operators.<br>b) Explain the uses of following functions:-<br>i) islower      ii) toupper      iii) tolower           | 05 |
|     | B) Attempt any one:<br>c) Write a program to read the strings.<br>d) Write a program to compute salesman's salary with suitable assuming data.   | 05 |

2017

Q.4

Fill in the blanks :

- i) Basic combined programming language is developed by ----- in 1967.
- ii) In C program, the smallest individual units are known as -----
- iii) The ----- contains the format of data being received.
- iv) The relational operator  $\leq$  means -----
- v) The operator ----- adds 1 to the operand, while ----- subtracts 1.

10

## SUBJECT CODE NO:- Y-2171

## FACULTY OF SCIENCE

B.Sc. T.Y (Sem-V) Examination March/April 2017

Mathematics MAT - 501 Real Analysis -I (Revised)

[Time: 1:30 Hours]

[Max.Marks:50]

Please check whether you have got the right question paper.

- N.B
- All questions are compulsory.
  - Figures to the right indicate full marks.

- Q.1
- A) Attempt any one: 08
- If  $f: A \rightarrow B$  and if  $XCB, YCB$ , then prove that:  
 $f^{-1}(XUY) = f^{-1}(X)Uf^{-1}(Y)$
  - If  $\{S_n\}_{n=1}^{\infty}$  is a sequence of nonnegative numbers and if  $\lim_{n \rightarrow \infty} S_n = L$  then prove that  $L \geq 0$
- B) Attempt any one. 07
- If  $S$  is a universal set, ACS, BCS. Define the characteristic function of  $A$  and prove that:  
 $\chi_{A \cup B} = \max(\chi_A, \chi_B)$   
 $\chi_{A \cap B} = \min(\chi_A, \chi_B)$
  - Consider  $\{S_n\}_{n=1}^{\infty}$ , where  $S_n = \frac{1}{n}$ , for  $n=1, 2, 3, \dots$ , then prove that :  
 $\lim_{n \rightarrow \infty} S_n = 0$
- Q.2
- A) Attempt any one. 08
- Show that the sequence  $\left\{\left(1 + \frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$  is convergent.
  - If  $\{t_n\}_{n=1}^{\infty}$  is a sequence of real numbers, if  $\lim_{n \rightarrow \infty} t_n = M$ , where  $M \neq 0$ , then prove that  
 $\lim_{n \rightarrow \infty} \left(\frac{1}{t_n}\right) = \frac{1}{M}$
- B) Attempt any one. 07
- Using  $\epsilon - \delta$  method, prove that  
 $\lim_{n \rightarrow \infty} \frac{n^2}{(n-7)^2 - 6} = 1$
  - If  $u^3 + v^3 = x + y, u^2 + v^2 = x^3 + y^3$ , show that  
 $\frac{\partial(u, v)}{\partial(x, y)} = \frac{1}{2} \frac{y^2 - x^2}{uv(u - v)}$
- Q.3
- A) Attempt any one. 05
- If  $\sum_{n=1}^{\infty} a_n$  converges to  $A$ , and  $\sum_{n=1}^{\infty} b_n$  converges to  $B$ , then prove that  
 $\sum_{n=1}^{\infty} (a_n + b_n)$  converges to  $A + B$ .
  - Show that the series  $\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)$  is divergent
- B) Attempt any one. 05

c) Prove that the series  $\sum_{n=1}^{\infty} \left[ \frac{1}{n(n+1)} \right]$  converges.

05

d) Using ratio test, show that the series  $\sum_{n=1}^{\infty} \frac{n^n}{n!}$  diverges

05

Q.4 Choose correct alternative.

10

i) If  $f(x) = x^2$  ( $-\infty < x < \infty$ ), then  $f^{-1}(7)$  is

- a)  $\{7\}$
- b)  $\{-7\}$
- c)  $\{0\}$
- d)  $\phi$

ii) If B is a infinite subset of the countable set A, than B is

- a) Uncountable
- b) Countable
- c) Equivalence to A
- d) None of these

iii)  $\lim_{n \rightarrow \infty} \left( 1 + \frac{2}{n} \right)^n$  is

- a) e
- b)  $e^2$
- c)  $e^n$
- d) 0

iv) The series  $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots$  is

- a) Non decreasing
- b) Non increasing
- c) Alternating
- d) None of these

v) Let  $\{S_n\}_{n=1}^{\infty}$  converges to  $L \neq 0$  then the sequence  $\{(-1)^n S_n\}_{n=1}^{\infty}$  is

- a) Oscillate
- b) Converges
- c) Diverges to  $\infty$
- d) Diverges to  $-\infty$

## SUBJECT CODE NO:- Y-2173

## FACULTY OF SCIENCE

B.Sc. T.Y (Sem-V) Examination March/April 2017

Mathematics MAT - 502 Abstract Algebra - I (Revised)

[Time: 1:30 Hours]

[Max.Marks:50]

Please check whether you have got the right question paper.

N.B

- i. Attempt all questions.
- ii. Figures to the right indicate full marks.

- Q.1 A) Attempt any one: 08
- a) If  $G$  is a group then prove that following
    - i) The identify element of  $G$  is unique
    - ii) Every  $a \in G$  has a unique inverse in  $G$
    - iii) For every  $a \in G$ ,  $(a^{-1})^{-1} = a$
    - iv) For all  $a, b \in G$ ,  $(a \cdot b)^{-1} = b^{-1}a^{-1}$
  - b) Prove that  $I(G) \approx G/Z$ , where  $I(G)$  is the group of inner automorphism of  $G$  and  $Z$  is centre of  $G$ .
- B) Attempt any one. 07
- c) If  $H$  and  $K$  are finite subgroups of  $G$  of order  $O(H)$  and  $O(K)$  respectively then prove that
 
$$O(HK) = \frac{O(H)O(K)}{O(H \cap K)}$$
  - d) Define normal subgroup and show that  $N$  is a normal subgroup of  $G$  if and only if  $g^N g^{-1} = N$  for all  $g \in G$
- Q.2 A) Attempt any one. 08
- a) If  $R$  be a commutative ring with unit element whose only ideals are  $(0)$  and  $R$  itself then prove that  $R$  is a field.
  - b) If  $f(x), g(x)$  are two nonzero element of  $F[x]$  then prove that
 
$$\deg(f(x) \cdot g(x)) = \deg f(x) + \deg g(x)$$
- B) Attempt any one. 07
- c) If every  $x \in R$  satisfies  $x^2 = x$  prove that  $R$  must be commutative ring.
  - d) Define commutative ring and prove the following for all  $a, b \in R$ 
    - i)  $a \cdot o = o \cdot a = 0$
    - ii)  $a(-b) = (-a)b = -(ab)$
    - iii)  $(-a)(-b) = ab$
- Q.3 A) Attempt any one. 05
- a) Show that the relation  $a \equiv b \pmod{H}$  is an equivalence relation.
  - b) If  $H$  is a subgroup of  $G$  and  $a \in G$ . Let  $aHa^{-1} = \{aha^{-1} | h \in H\}$  then prove that  $aHa^{-1}$  is a subgroup of  $G$ .

B) Attempt any one.

- c) Prove that  $H \cap K$  is nontrivial when  $o(H) > \sqrt{o(G)}$ ,  $o(K) > \sqrt{o(G)}$
- d) Prove that  $H \cap K$  is normal subgroup of  $H$  where  $H$  is a subgroup of  $G$  and  $N$  is a normal subgroup of  $G$ .

05

Q.4 Choose correct alternative.

10

- i)  $a \equiv b \pmod{H}$ , if -----.
  - a)  $ab \in H$
  - b)  $ba \in H$
  - c)  $ab^{-1} \in H$
  - d) None of these
- ii) If  $P/o(G)$ ,  $P$ -prime then -----.
  - a)  $G$  has a subgroup of order  $P$ .
  - b)  $G$  need not have a subgroup of order  $P$ .
  - c)  $G$  has no element of order  $P$ .
  - d) None of these
- iii) If  $G/N$  is abelian then  $G$  is -----.
  - a) Abelian
  - b) Need not be abelian
  - c) Cyclic
  - d) Noncyclic
- iv) Which of the following ring is non-commutative
  - a)  $\langle \mathbb{Z}, +, . \rangle$
  - b)  $\langle \mathbb{R}, +, . \rangle$
  - c)  $\langle M_{2 \times 2}(\mathbb{R}), +, . \rangle$
  - d)  $\langle \mathbb{Q}, +, . \rangle$
- v) Which of the following polynomial is irreducible over the field of integer moduls 2?
  - a)  $x^2 - 1$
  - b)  $(x^2 - 1)^2$
  - c)  $x^2 + x + 1$
  - d) None of these

**SUBJECT CODE NO:- Y-2015**  
**FACULTY OF SCIENCE**  
**B.Sc. T.Y (Sem-VI) Examination March/April 2017**  
**Mathematics MAT - 602 (Revised)**  
**Abstract Algebra - II**

[Time: 1:30 Hours]

[Max.Marks:50]

Please check whether you have got the right question paper.

- N.B
- 1) All questions are compulsory.
  - 2) Figures to the right indicate full marks.

- Q.1 A) Attempt any one: 08
- a) If  $T$  is homomorphism of a vector space  $U$  onto a vector space  $V$  over a field  $F$  with kernel  $W$ , then prove that  $V$  is isomorphic to  $U/W$ .
  - b) If  $\vartheta_1, \vartheta_2, \dots, \vartheta_n \in V$  are linearly independent, where  $V$  is vector space over field  $F$ , then prove that every element in their span has a unique representation in the form  $\lambda_1\vartheta_1 + \lambda_2\vartheta_2 + \dots + \lambda_n\vartheta_n$  with  $\lambda_i \in F$ .
- B) Attempt any one: 07
- a) Show that in  $F^{(3)}$  the vectors  $(1,1,0), (3,1,3), (5,3,3)$  are linearly dependent.
  - b) If  $F$  is a field of real numbers and if  $V$  is a set of all sequence of the form  $(a_1, a_2, \dots, a_n, \dots)$ ,  $a_i \in F$ , where equality, addition and scalar multiplication are defined component wise. If  $W = \{(a_1, a_2, \dots, a_n, \dots) \in V \mid \lim_{n \rightarrow \infty} a_n = 0\}$ . Prove that  $W$  is a subspace of  $V$ .
- Q.2 A) Attempt any one: 08
- a) Let  $V$  be a finite – dimensional inner product space, then prove that  $V$  has an orthonormal set as basis.
  - b) If  $V$  is finite –dimensional vector space over a field  $F$  and  $W$  is a subspace of  $V$ , then prove that  $A(A(W)) = W$ .
- B) Attempt any one: 07
- c) In the vector space  $F^{(n)}$  for the vectors  $u = (\alpha_1, \alpha_2, \dots, \alpha_n)$  and  $\vartheta = (\beta_1, \beta_2, \dots, \beta_n)$ , if  $(u, \vartheta) = \alpha_1\overline{\beta_1} + \alpha_2\overline{\beta_2} + \dots + \alpha_n\overline{\beta_n}$  then show that this defines an inner product on  $F^{(n)}$ .
  - d) If  $F$  is the real field and  $V$  is  $F^{(3)}$ , inner product space, show that Schwartz inequality implies that the cosine of an angle is of absolute value at most 1.
- Q.3 A) Attempt any one: 05
- a) If  $\vartheta_1, \vartheta_2, \dots, \vartheta_n$  are in a vector space over the field  $F$  then prove that either they are linearly independent or some  $\vartheta_k$  is linear combination of preceding ones  $\vartheta_1, \vartheta_2, \dots, \vartheta_{k-1}$ .
  - b) If  $W$  is a subspace of a vector space  $V$  over the field  $F$  then prove that  $A(W)$  is subspace of  $\hat{V}$ .
- B) Attempt any one: 05
- c) If  $A$  and  $B$  are sub modules of module  $M$ , prove that  $A \cap B$  is a submodule of  $M$ .
  - d) If  $V$  is a finite-dimensional vector space over field  $F$  and  $\vartheta_1 \neq \vartheta_2$  are in  $V$ , prove that there is an element  $f \in \hat{V}$  such that  $f(\vartheta_1) \neq f\vartheta_2$ .

Q.4

Choose the correct alternative:

10

- 1) If  $W$  is a subspace of  $n$ -dimensional vector space  $V$  over field  $F$  then-----
  - a)  $\dim W = n$
  - b)  $\dim W < n$
  - c)  $\dim W \leq n$
  - d)  $\dim W \geq n$
- 2) The length of a vector  $u$  is given by  $\|u\|$ -----
  - a)  $(u, u)$
  - b)  $\sqrt{(u, u)}$
  - c)  $(u, u)^2$
  - d)  $|(u, u)|$
- 3) The number of elements in any basis of a vector space  $R^{(4)}(R)$  is -----
  - a) 1
  - b) 2
  - c) 3
  - d) 4
- 4) An orthogonal set of non-zero vectors is -----
  - a) Linearly dependent
  - b) Orthonormal set
  - c) Basis
  - d) Linearly independent
- 5) Every field is a vector space over -----
  - a) Itself
  - b) Vector space
  - c) Module
  - d) Ring



**SUBJECT CODE NO:- Y-2013**  
**FACULTY OF SCIENCE**  
**B.Sc. T.Y (Sem-VI) Examination March/April 2017**  
**Mathematics MAT-601 (Revised)**  
**Real Analysis-II**

[Time: 1:30 Hours]

[Max.Marks:50]

Please check whether you have got the right question paper.

- N.B
- i) All questions are compulsory.
  - ii) Figures to right indicate full marks.

- Q.1 Attempt any one: 08
- A) a) If  $F_1$  and  $F_2$  are closed subset of the metric space  $M$ , then prove that  $F_1 \cup F_2$  is also closed subset of  $M$ .  
 b) Prove that the metric space  $(M, P)$  is compact if and only if every sequence of points of  $M$  has a sequence of points of  $M$  converging to a point in  $M$ .
- B) Attempt any one: 07
- a) If  $l^1$  is the class of all sequences  $\{\$n\}_{n=1}^{\infty}$  of real numbers such that  $\sum_{n=1}^{\infty} |\$n| < \infty$ . If  $\$ = \{\$n\}_{n=1}^{\infty}$  and  $t = \{t_n\}_{n=1}^{\infty}$  are in  $l^1$ , show that  $P(\$ , t) = \sum_{n=1}^{\infty} |\$n - t_n|$  defines metric for  $l^1$ .  
 b) Prove that the internal  $(0,1)$  with the absolute value metric is not a complete metric space.
- Q.2A) Attempt any one: 08
- a) If  $f \in \mathcal{H}[a, c]$  and  $a < c < b$  then prove that  
 $f \in \mathcal{H}[a, c], f \in \mathcal{H}[c, b]$  and  $\int_a^b f = \int_a^c f + \int_c^b f$ .  
 b) Prove that every open subset  $G$  of  $\mathbb{R}^1$  can be written as  $G = \cup I_n$ , where  $I_1, I_2, \dots$  are finite or countable number of intervals which are mutually disjoint.
- B) Attempt any one: 07
- c) If  $A$  and  $B$  are open subsets of  $\mathbb{R}^1$ , prove that  $A \times B$  is open subset of  $\mathbb{R}^2$   
 d) Find the Fourier series for the function  $f(x) = e^x$  in  $[-\Pi, \Pi]$
- Q.3A) Attempt any one: 05
- a) If  $f(x)$  is expanded in series of sines in the form of  $f(x) = \sum_{n=1}^{\infty} b_n \sin nx$ , then prove that :  
 $b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx \, dx$ .  
 b) If  $f'(x) = g'(x)$  for all  $x$  in the closed bounded interval  $[a, b]$ , then prove that  $f - g$  is constant function.
- B) Attempt any one: 05
- c) Prove that :  
 $\frac{2\pi^2}{9} \leq \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{2x}{\sin x} \, dx \leq \frac{4\pi^2}{9}$ .  
 d) If  $T: X \rightarrow X$  is defined as  $T_x = x^2$  for all  $x \in [0, \frac{1}{3}]$ . Prove that  $T$  is contraction on  $[0, \frac{1}{3}]$

Q.4 Choose the correct alternative:

- i. The metric defined by  $P(x, y) = |x - y|$  for all  $x, y \in \mathbb{R}$ , the set of real number is called as .....
  - a. Discrete metric
  - b. Indiscrete metric
  - c. Absolute value metric
  - d. None of these.
- ii. In any metric space  $(M, \rho)$ ; M and the empty set  $\emptyset$  are .....
  - a. Only open
  - b. Only closed
  - c. Both open and closed
  - d. None of these.
- iii. If A and B are closed subsets of a metric space M. If  $A \subseteq B$  then .....
  - a.  $\bar{A} \subseteq \bar{B}$
  - b.  $\bar{B} \subseteq \bar{A}$
  - c.  $\bar{A} = \bar{B}$
  - d.  $\bar{A} = A$ .
- iv. If  $f$  is a bounded function on the closed and the bounded interval  $[a, b]$ , if  $\sigma$  is any subdivision of  $[a, b]$  then  $\int_a^b f(x) dx = \dots$ 
  - a.  $l.u.b. L[f, \sigma]$
  - b.  $l.u.b. U[f, \sigma]$
  - c.  $g.l.b. U[f, \sigma]$
  - d.  $g.l.b. L[f, \sigma]$ .
- v. A function  $f$  is said to be odd function if .....
  - a.  $f(-x) = f(x)$
  - b.  $f(-x) = -f(x)$
  - c.  $f(x) = f(x^2)$
  - d.  $f(x) = f(x^3)$ .